Programming Assignment 4: Paths in Graphs

Revision: June 19, 2018

Introduction

Welcome to your fourth programming assignment of the Graph Algorithms course! In this assignments we focus on shortest paths in weighted graphs.

Learning Outcomes

Upon completing this programming assignment you will be able to:

1. compute the minimum cost of a flight from one city to another one;
2. detect anomalies in currency exchange rates;
3. compute optimal way of exchanging the given currency into all other currencies.

Passing Criteria: 2 out of 3

Passing this programming assignment requires passing at least 2 out of 3 programming challenges from this assignment. In turn, passing a programming challenge requires implementing a solution that passes all the tests for this problem in the grader and does so under the time and memory limits specified in the problem statement.

Contents

Graph Representation in Programming Assignments

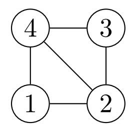
In programming assignments, graphs are given as follows. The first line contains non-negative integers n and m — the number of vertices and the number of edges respectively. The vertices are always numbered from 1 to n. Each of the following m lines defines an edge in the format `u v` where $1 \leq u, v \leq n$ are endpoints of the edge. If the problem deals with an undirected graph this defines an undirected edge between u and v. In case of a directed graph this defines a directed edge from u to v. If the problem deals with a weighted graph then each edge is given as `u v w` where u and v are vertices and w is a weight.

It is guaranteed that a given graph is simple. That is, it does not contain self-loops (edges going from a vertex to itself) and parallel edges.

Examples:

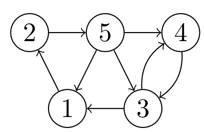
- An undirected graph with four vertices and five edges:

```
4 5
2 1
4 3
1 4
2 4
3 2
```



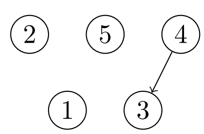
- A directed graph with five vertices and eight edges.

```
5 8
4 3
1 2
3 1
3 4
2 5
5 1
5 4
5 3
```



- A directed graph with five vertices and one edge.

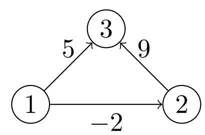
```
5 1
4 3
```



Note that the vertices 1, 2, and 5 are isolated (have no adjacent edges), but they are still present in the graph.

- A weighted directed graph with three vertices and three edges.

```
3 3
2 3 9
1 3 5
1 2 -2
```



1 Computing the Minimum Cost of a Flight

Problem Introduction

Now, you are interested in minimizing not the number of segments, but the total cost of a flight. For this you construct a weighted graph: the weight of an edge from one city to another one is the cost of the corresponding flight.

Problem Description

Task. Given an *directed* graph with positive edge weights and with n vertices and m edges as well as two vertices u and v, compute the weight of a shortest path between u and v (that is, the minimum total weight of a path from u to v).

Input Format. A graph is given in the standard format. The next line contains two vertices u and v.

Constraints. $1 \leq n \leq 10^4$, $0 \leq m \leq 10^5$, $u \neq v$, $1 \leq u, v \leq n$, edge weights are non-negative integers not exceeding 10^3.

Output Format. Output the minimum weight of a path from u to v, or -1 if there is no path.

Time Limits.

language	C	C++	Java	Python	Haskell	JavaScript	Scala
time (sec)	2	2	3	10	4	10	6

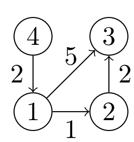
Sample 1.

Input:

```
4 4
1 2 1
4 1 2
2 3 2
1 3 5
1 3
```

Output:

```
3
```



There is a unique shortest path from vertex 1 to vertex 3 in this graph ($1 \to 2 \to 3$), and it has weight 3.

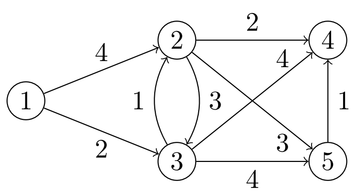
Sample 2.

Input:

```
5 9
1 2 4
1 3 2
2 3 2
3 2 1
2 4 2
3 5 4
5 4 1
2 5 3
3 4 4
1 5
```

Output:

```
6
```



There are two paths from 1 to 5 of total weight 6: $1 \to 3 \to 5$ and $1 \to 3 \to 2 \to 5$.

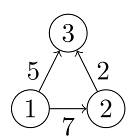
Sample 3.

Input:

```
3 3
1 2 7
1 3 5
2 3 2
3 2
```

Output:

```
-1
```



There is no path from 3 to 2.

Need Help?

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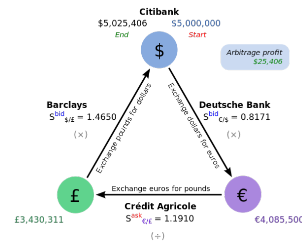
2 Detecting Anomalies in Currency Exchange Rates

Problem Introduction

You are given a list of currencies $c_1, c_2, \ldots, c_n$ together with a list of exchange rates: r_{ij} is the number of units of currency c_j that one gets for one unit of c_i. You would like to check whether it is possible to start with one unit of some currency, perform a sequence of exchanges, and get more than one unit of the same currency. In other words, you would like to find currencies $c_{i_1}, c_{i_2}, \ldots, c_{i_k}$ such that $r_{i_1,i_2} \cdot r_{i_2,i_3} \cdot r_{i_{k-1},i_k}, r_{i_k,i_1} > 1$. For this, you construct the following graph: vertices are currencies $c_1, c_2, \ldots, c_n$, the weight of an edge from c_i to c_j is equal to $-\log r_{ij}$. There it suffices to check whether is a negative cycle in this graph. Indeed, assume that a cycle $c_i \to c_j \to c_k \to c_i$ has negative weight. This means that $-(\log c_{ij} + \log c_{jk} + \log c_{ki}) < 0$ and hence $\log c_{ij} + \log c_{jk} + \log c_{ki} > 0$. This, in turn, means that



$$r_{ij} r_{jk} r_{ki} = 2^{\log c_{ij}} 2^{\log c_{jk}} 2^{\log c_{ki}} = 2^{\log c_{ij} + \log c_{jk} + \log c_{ki}} > 1\,.$$

Problem Description

Task. Given an directed graph with possibly negative edge weights and with n vertices and m edges, check whether it contains a cycle of negative weight.

Input Format. A graph is given in the standard format.

Constraints. $1 \leq n \leq 10^3$, $0 \leq m \leq 10^4$, edge weights are integers of absolute value at most 10^3.

Output Format. Output 1 if the graph contains a cycle of negative weight and 0 otherwise.

Time Limits.

language	C	C++	Java	Python	Haskell	JavaScript	Scala
time (sec)	2	2	3	10	4	10	6

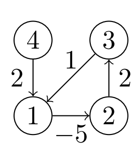
Sample 1.

Input:

```
4 4
1 2 -5
4 1 2
2 3 2
3 1 1
```

Output:

```
1
```



The weight of the cycle $1 \to 2 \to 3$ is equal to -2, that is, negative.

Need Help?

Ask a question or check out the questions asked by other learners at this forum thread.

3 Exchanging Money Optimally

Problem Introduction

Now, you would like to compute an optimal way of exchanging the given currency c_i into all other currencies. For this, you find shortest paths from the vertex c_i to all the other vertices.

Problem Description

Task. Given an directed graph with possibly negative edge weights and with n vertices and m edges as well as its vertex s, compute the length of shortest paths from s to all other vertices of the graph.

Input Format. A graph is given in the standard format.

Constraints. $1 \leq n \leq 10^3$, $0 \leq m \leq 10^4$, $1 \leq s \leq n$, edge weights are integers of absolute value at most 10^9.

Output Format. For all vertices i from 1 to n output the following on a separate line:

- "*", if there is no path from s to u;
- "-", if there is a path from s to u, but there is no shortest path from s to u (that is, the distance from s to u is $-\infty$);
- the length of a shortest path otherwise.

Time Limits.

language	C	C++	Java	Python	Haskell	JavaScript	Scala
time (sec)	2	2	3	10	4	10	6

Sample 1.

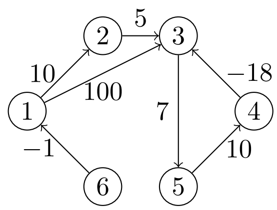
Input:

```
6 7
1 2 10
2 3 5
1 3 100
3 5 7
5 4 10
4 3 -18
6 1 -1
1
```

Output:

```
0
10
-
-
-
*
```

Explanation:



The first line of the output states that the distance from 1 to 1 is equal to 0. The second one shows that the distance from 1 to 2 is 10 (the corresponding path is $1 \to 2$). The next three lines indicate that the distance from 1 to vertices 3, 4, and 5 is equal to $-\infty$: indeed, one first reaches the vertex 3 through edges $1 \to 2 \to 3$ and then makes the length of a path arbitrary small by making sufficiently many walks through the cycle $3 \to 5 \to 4$ of negative weight. The last line of the output shows that there is no path from 1 to 6 in this graph.

Sample 2.

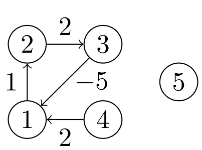
Input:

```
5 4
1 2 1
4 1 2
2 3 2
3 1 -5
4
```

Output:

```
-
-
-
0
*
```

Explanation:



In this case, the distance from 4 to vertices 1, 2, and 3 is $-\infty$ since there is a negative cycle $1 \to 2 \to 3$ that is reachable from 4. The distance from 4 to 4 is zero. There is no path from 4 to 5.

Need Help?

Ask a question or check out the questions asked by other learners at this forum thread.