دانشکده مهندسی کامپیوتر

هوش مصنوعی و سیستم‌های خبره

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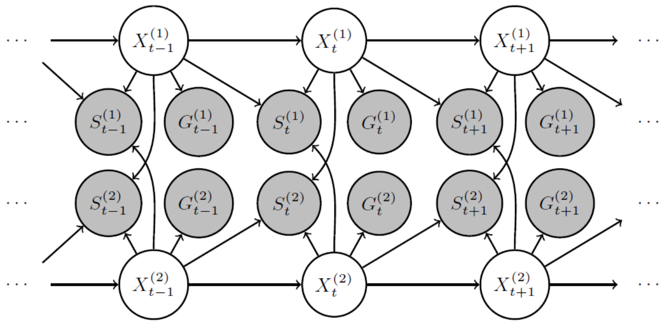
[1] در طراحی این تمرین از منابع کورس CS188 دانشگاه برکلی استفاده شده است.

۱ Particle Filtering: Where are the Two Cars?

As before, we are trying to estimate the location of cars in a city, but now, we model two cars jointly, i.e. car i for $i \in \{1, 2\}$. The modified HMM model is as follows:

- $X^{(i)}$– the location of car i
- $S^{(i)}$– the noisy location of the car i from the signal strength at a nearby cell phone tower
- $G^{(i)}$– the noisy location of car i from GPS



d	$D(d)$	$E_L(d)$	$E_N(d)$	$E_G(d)$
-4	0.05	0	0.02	0
-3	0.10	0	0.04	0.03
-2	0.25	0.05	0.09	0.07
-1	0.10	0.10	0.20	0.15
0	0	0.70	0.30	0.50
1	0.10	0.10	0.20	0.15
2	0.25	0.05	0.09	0.07
3	0.10	0	0.04	0.03
4	0.05	0	0.02	0

The signal strength from one car gets noisier if the other car is at the same location. Thus, the observation $S^{(i)}_t$ also depends on the current state of the other car $X^{(j)}_t$, $j \neq i$.

The transition is modeled using a drift model D, the GPS observation $G^{(i)}_t$ using the error model E_G, and the observation $S^{(i)}_t$ using one of the error models E_L or E_N, depending on the car's speed and the relative location of both cars. These drift and error models are in the table above. **The transition and observation models are:**

$$P(X^{(i)}_t | X^{(i)}_{t-1}) = D(X^{(i)}_t - X^{(i)}_{t-1})$$

$$P(S^{(i)}_t | X^{(i)}_{t-1}, X^{(i)}_t, X^{(j)}_t) = \begin{cases} E_N(X^{(i)}_t - S^{(i)}_t), & \text{if } |X^{(i)}_t - X^{(i)}_{t-1}| \geq 2 \text{ or } X^{(i)}_t = X^{(j)}_t \\ E_L(X^{(i)}_t - S^{(i)}_t), & \text{otherwise} \end{cases}$$

$$P(G^{(i)}_t | X^{(i)}_t) = E_G(X^{(i)}_t - G^{(i)}_t).$$

Throughout this problem you may give answers either as unevaluated numeric expressions (e.g. $0.1 \cdot 0.5$) or as numeric values (e.g. 0.05). The questions are decoupled.

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Assume that at t = 3, we have the single particle $(X^{(1)}_3 = -1, X^{(2)}_3 = 2)$.

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What is the probability that this particle becomes $(X^{(1)}_4 = -3, X^{(2)}_4 = 3)$ after passing it through the dynamics model?

Answer= ....

Your Solution:

پاسخ:

$$\begin{aligned} P(X_4^{(1)} = -3, X_4^{(2)} = 3 | X_3^{(1)} = -1, X_3^{(2)} = 2) &= P(X_4^{(1)} = -3 | X_3^{(1)} = -1) \cdot P(X_4^{(2)} = 3 | X_3^{(2)} = 2) \\ &= D(-3-(-1)) \cdot D(3-2) \\ &= 0.25 \cdot 0.10 \\ &= 0.025 \end{aligned}$$

Answer: 0.025

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Assume that there are no sensor readings at t = 4. What is the joint probability that the *original* single particle (from t = 3) becomes $(X^{(1)}_4 = -3, X^{(2)}_4 = 3)$ and then becomes $(X^{(1)}_5 = -4, X^{(2)}_5 = 4)$?

Answer= ....

Your Solution:

For the remaining of this problem, we will be using 2 particles at each time step.

پاسخ:

$$
\begin{aligned}
&P(X_4^{(1)} = -3, X_5^{(1)} = -4, X_4^{(2)} = 3, X_5^{(2)} = 4 | X_3^{(1)} = -1, X_3^{(2)} = 2) \\
&= P(X_4^{(1)} = -3, X_5^{(1)} = -4 | X_3^{(1)} = -1) \cdot P(X_4^{(2)} = 3, X_5^{(2)} = 4 | X_3^{(2)} = 2) \\
&= P(X_5^{(1)} = -4 | X_4^{(1)} = -3) \cdot P(X_4^{(1)} = -3 | X_3^{(1)} = -1) \cdot P(X_5^{(2)} = 4 | X_4^{(2)} = 3) \cdot P(X_4^{(2)} = 3 | X_3^{(2)} = 2) \\
&= D(-4 - (-3)) \cdot D(-3 - (-1)) \cdot D(4 - 3) \cdot D(3 - 2) \\
&= 0.10 \cdot 0.25 \cdot 0.10 \cdot 0.10 \\
&= 0.00025
\end{aligned}
$$

Answer: 0.00025

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At t = 6, we have particles $[(X^{(1)}_6 = 3, X^{(2)}_6 = 0), (X^{(1)}_6 = 3, X^{(2)}_6 = 5)]$. Suppose that after weighting, resampling, and transitioning from t = 6 to t = 7, the particles become $[(X^{(1)}_7 = 2, X^{(2)}_7 = 2), (X^{(1)}_7 = 4, X^{(2)}_7 = 1)]$.

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Suppose both cars' cell phones died so you only get the observations $G^{(1)}_7 = 2, G^{(2)}_7 = 2$. What is the weight of each particle?

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	

Your Solution:

پاسخ:

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	$P(G_7^{(1)} = 2 \vert X_7^{(1)} = 2) \cdot P(G_7^{(2)} = 2 \vert X_7^{(2)} = 2)$ $= E_G(2-2) \cdot E_G(2-2)$ $= 0.50 \cdot 0.50$ $= 0.25$
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	$P(G_7^{(1)} = 2 \vert X_7^{(1)} = 4) \cdot P(G_7^{(2)} = 2 \vert X_7^{(2)} = 1)$ $= E_G(4-2) \cdot E_G(1-2)$ $= 0.07 \cdot 0.15$ $= 0.0105$

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To decouple this question, assume that you got the following weights for the two particles.

Particle	Weight
$(X_7^{(1)} = 2, X_7^{(2)} = 2)$	0.09
$(X_7^{(1)} = 4, X_7^{(2)} = 1)$	0.01

What is the belief for the location of car 1 and car 2 at t = 7?

Location	$P(X_7^{(1)})$	$P(X_7^{(2)})$
$X_7^{(i)} = 1$		
$X_7^{(i)} = 2$		
$X_7^{(i)} = 4$		

Your Solution:

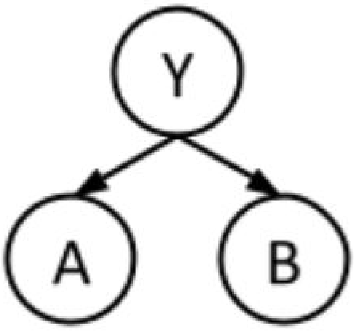
پاسخ:

Location	$P(X_7^{(1)})$	$P(X_7^{(2)})$
$X_7^{(i)} = 1$	$\frac{0}{0.09+0.01} = 0$	$\frac{0.01}{0.09+0.01} = 0.1$
$X_7^{(i)} = 2$	$\frac{0.09}{0.09+0.01} = 0.9$	$\frac{0.09}{0.09+0.01} = 0.9$
$X_7^{(i)} = 4$	$\frac{0.01}{0.09+0.01} = 0.1$	$\frac{0}{0.09+0.01} = 0$

۲ Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y , A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A	1	1	1	1	0	1	0	1	1	1
B	1	0	0	1	1	1	1	0	1	1
Y	1	1	0	0	0	1	1	0	0	0



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What are the maximum likelihood estimates for the tables P(Y), P(A|Y), and P(B|Y)?

Y	$P(Y)$
0	3/5
1	2/5

A	Y	$P(A\|Y)$
0	0	1/6
1	0	5/6
0	1	1/4
1	1	3/4

B	Y	$P(B\|Y)$
0	0	1/3
1	0	2/3
0	1	1/4
1	1	3/4

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Consider a new data point (A = 1, B = 1). What label would this classifier assign to this sample?

$$
\begin{aligned}
P(Y=0, A=1, B=1) &= P(Y=0)P(A=1|Y=0)P(B=1|Y=0) && (1)\\
&= (3/5)(5/6)(2/3) && (2)\\
&= 1/3 && (3)\\
P(Y=1, A=1, B=1) &= P(Y=1)P(A=1|Y=1)P(B=1|Y=1) && (4)\\
&= (2/5)(3/4)(3/4) && (5)\\
&= 9/40 && (6)\\
& && (7)
\end{aligned}
$$

Our classifier will predict label 0.

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Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for P(A|Y) given Laplace Smoothing with k = 2.

A	Y	$P(A\|Y)$
0	0	3/10
1	0	7/10
0	1	3/8
1	1	5/8