Decomposition of Graphs: Previsit and Postvisit Orders

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Graph Algorithms
Data Structures and Algorithms

Learning Objectives

- Compute the preorder and postorder numbers for a DFS.
- Understand why these numbers might be important.

Outline

Need to Record Data

- Plain `DFS` just marks all vertices as visited.
- Need to keep track of other data to be useful.
- Augment functions to store additional information.

Previsit and Postvisit Functions

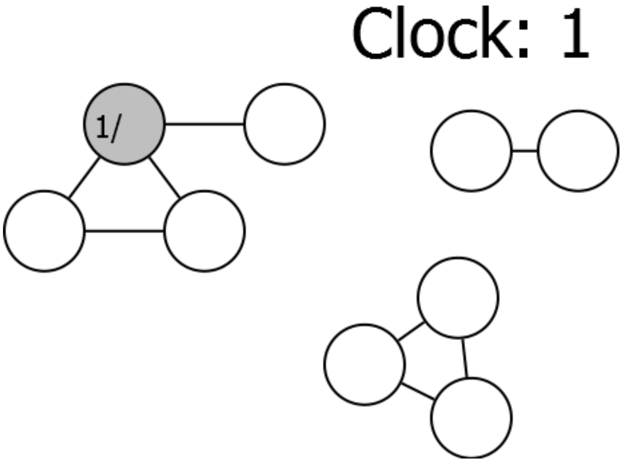
Explore(v)

```
visited(v) ← true
previsit(v)
for (v, w) ∈ E:
  if not visited(w):
    explore(w)
postvisit(v)
```

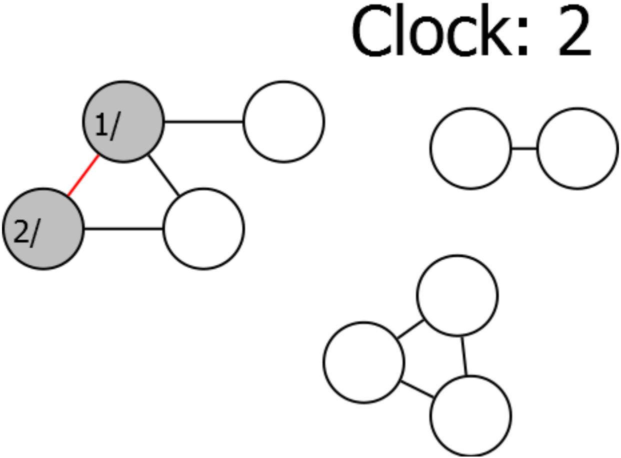
Clock

- Keep track of order of visits.
- Clock ticks at each pre-/post- visit.
- Records previsit and postvisit times for each v.

Example



Example



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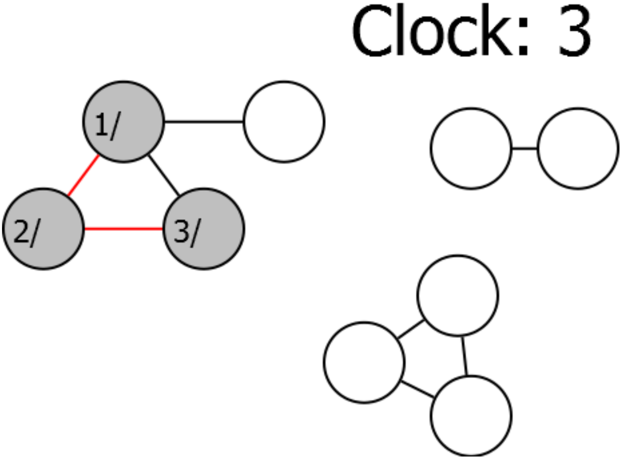

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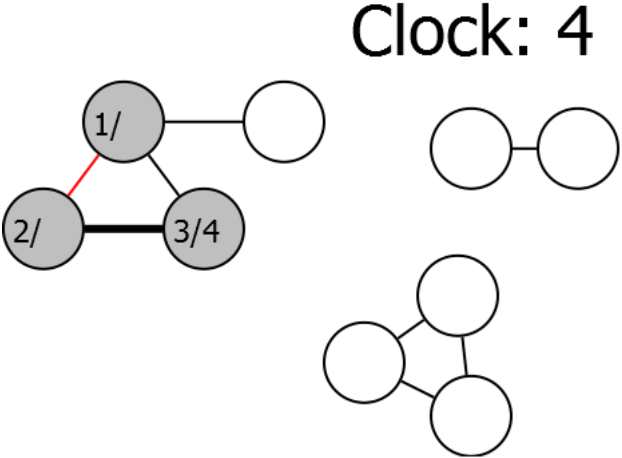

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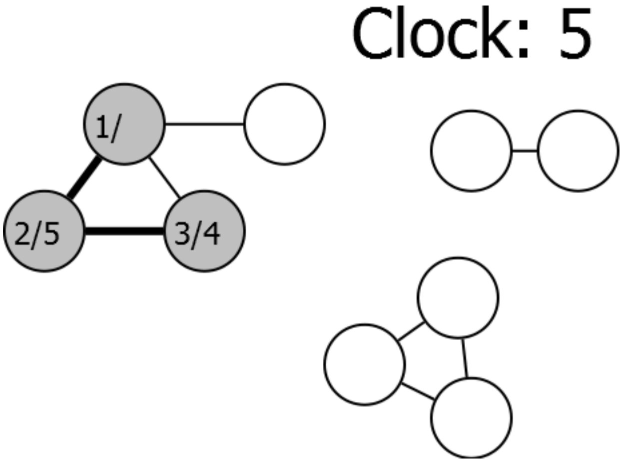

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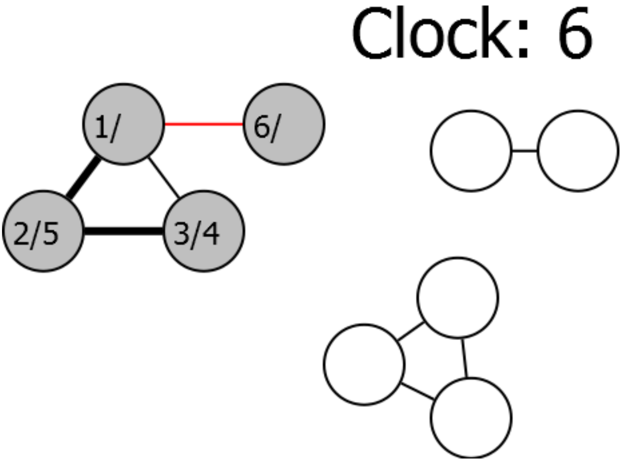

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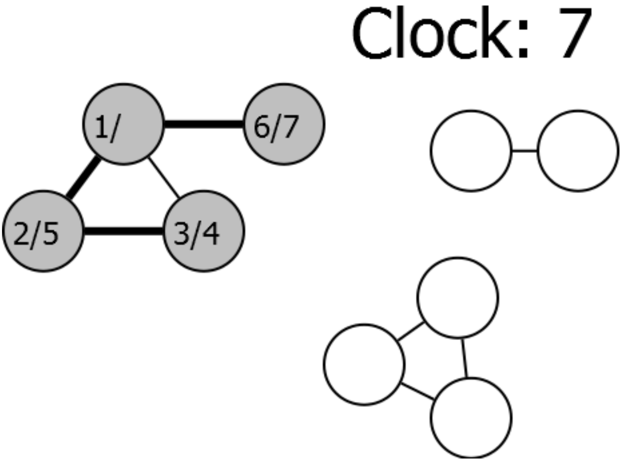

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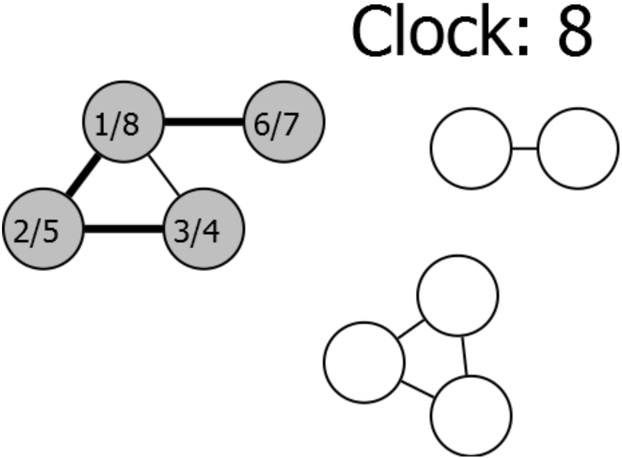

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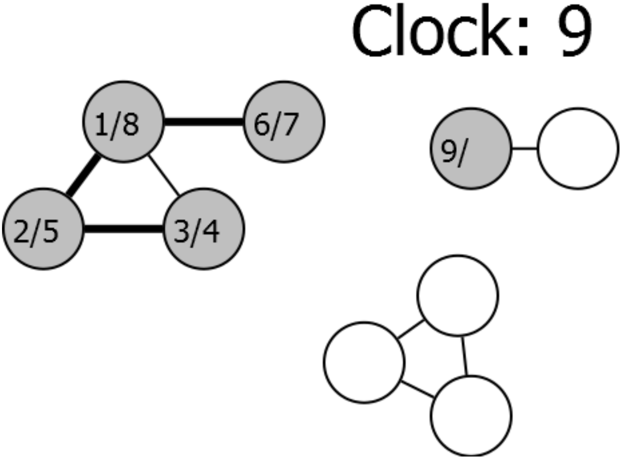

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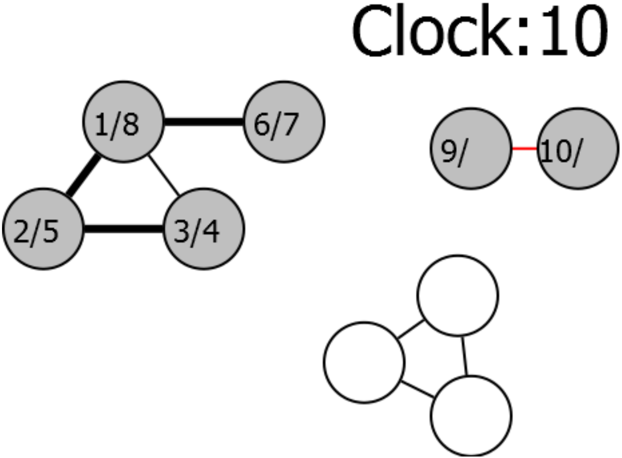

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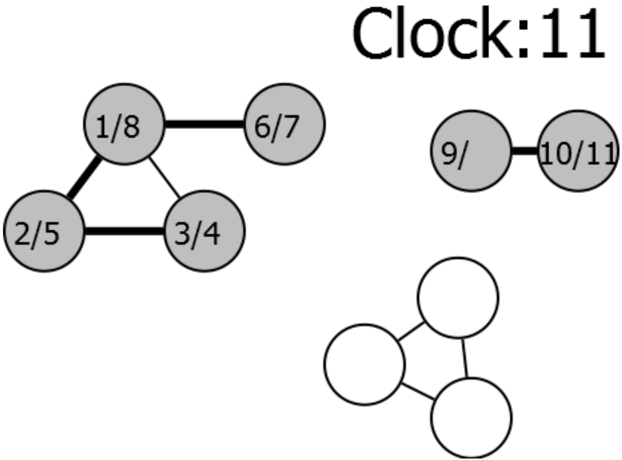

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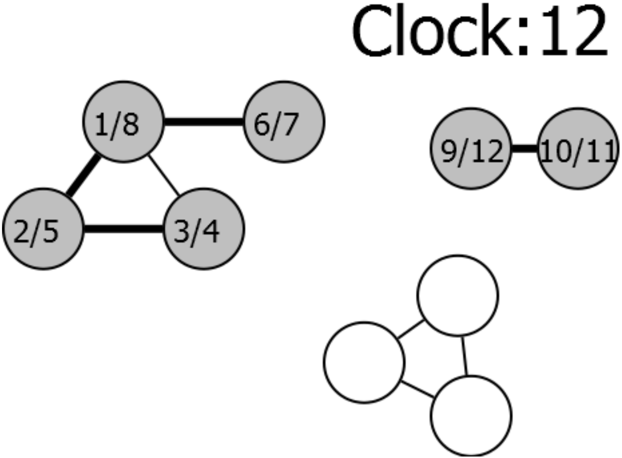

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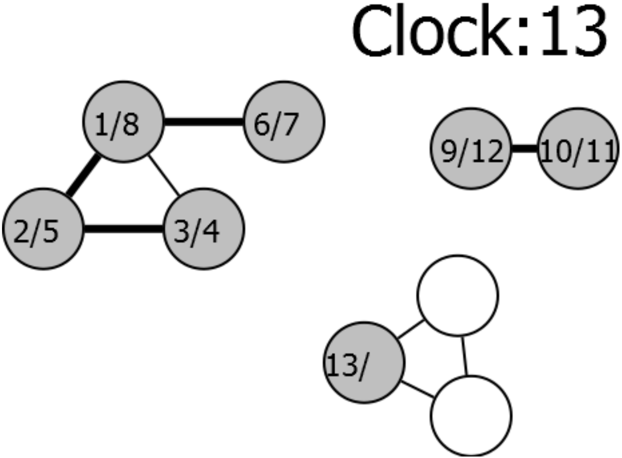

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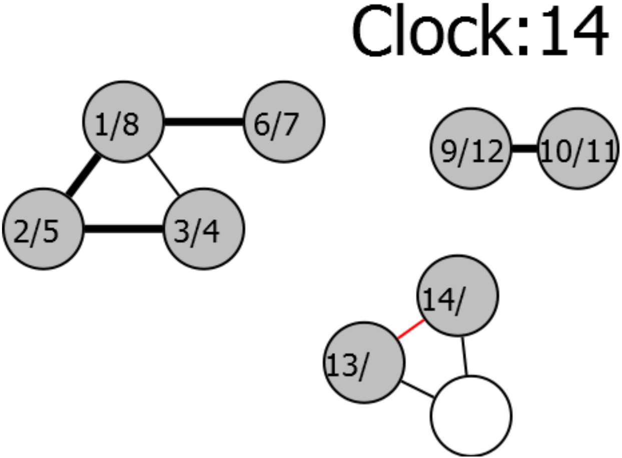

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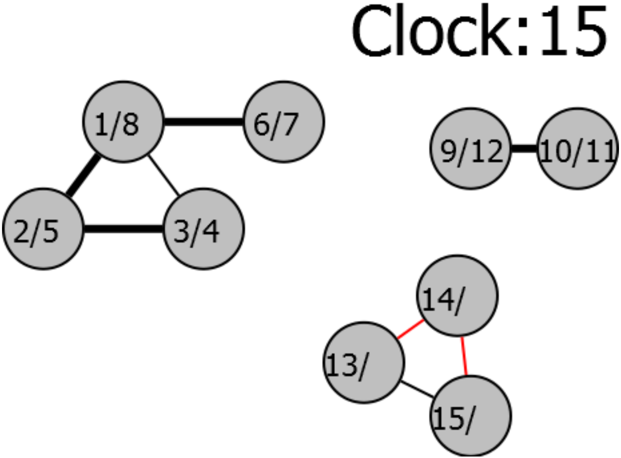

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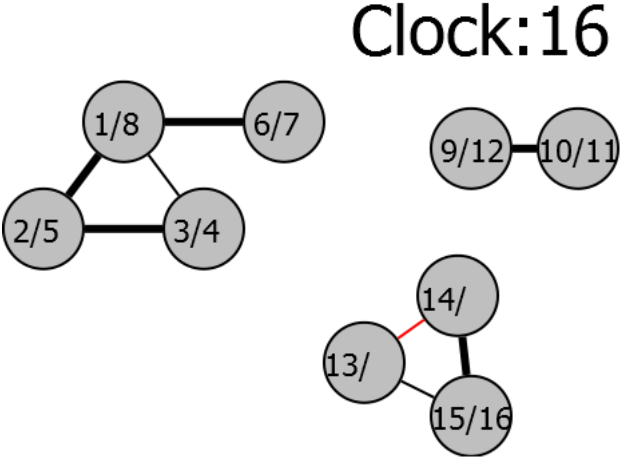

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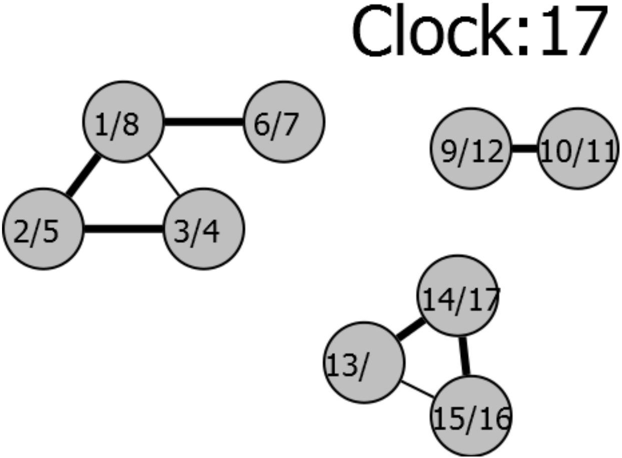

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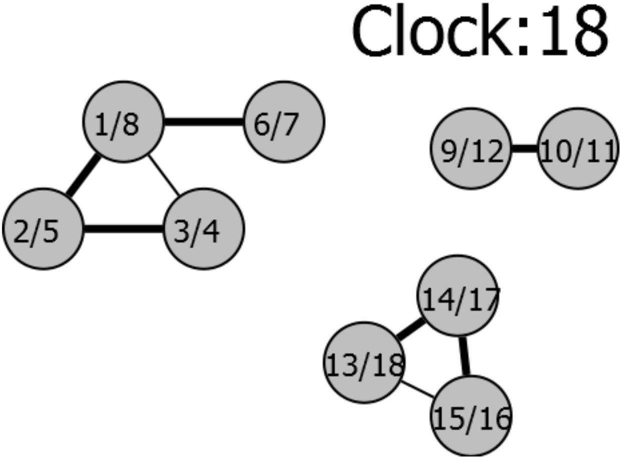


Computing Pre- and Post- Numbers

Initialize `clock` to 1.

previsit(v)

$\mathtt{pre}(v) \leftarrow \mathtt{clock}$

$\mathtt{clock} \leftarrow \mathtt{clock} + 1$

postvisit(v)

$\mathtt{post}(v) \leftarrow \mathtt{clock}$

$\mathtt{clock} \leftarrow \mathtt{clock} + 1$

Outline

Result

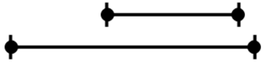
Previsit and Postvisit numbers tell us about the execution of DFS.

Lemma

For any vertices u, v the intervals $[\mathtt{pre}(u), \mathtt{post}(u)]$ and $[\mathtt{pre}(v), \mathtt{post}(v)]$ are either nested or disjoint.

Explanation

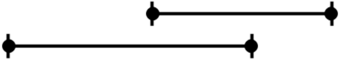
Nested



Disjoint

Explanation

Interleaved (not possible)



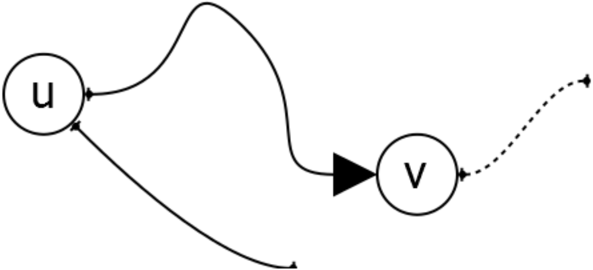
Proof

Assume that u visited before v. Two cases

- Find v while exploring u (u an ancestor of v)
- Find v after exploring u (u a cousin of v)

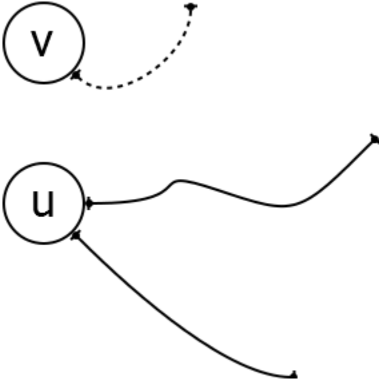
Case I

If you explore v while exploring u, cannot finish exploring u until done exploring v. Therefore nested.



Case II

If explore v after finish exploring u, $\texttt{post}(u) < \texttt{pre}(v)$, therefore disjoint.



Problem

Which of the following tables is not a valid set of pre- and post- orders?

Vert.	Pre	Post
A	1	8
B	9	10
C	3	4
D	2	7
E	5	6

Vert.	Pre	Post
A	1	9
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Solution

Which of the following tables is not a valid set of pre- and post- orders?

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Next Time

Directed graphs.