Decomposition of Graphs: Connectivity

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Graph Algorithms
Data Structures and Algorithms

Learning Objectives

- Understand the importance of connected components of a graph.
- Compute the connected components of a graph.

Outline

Reachability

Want to understand which vertices in G are reachable from which others.

Connected Components

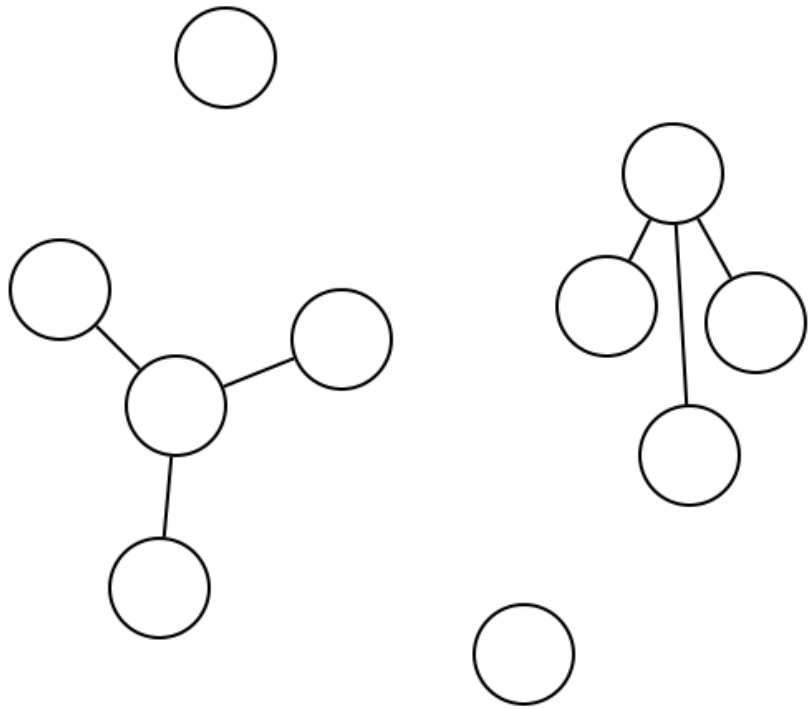
Theorem

The vertices of a graph G can be partitioned into Connected Components so that v is reachable from w if and only if they are in the same connected component.

Problem

How many connected components does the graph below have?

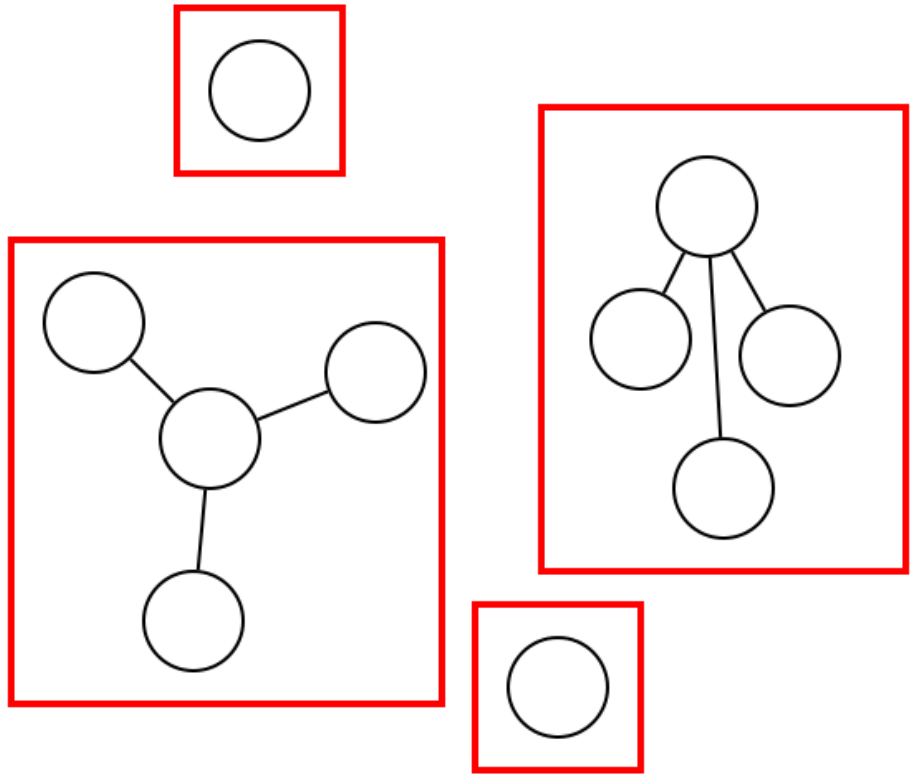
.



Solution

How many connected components does the graph below have?

4.



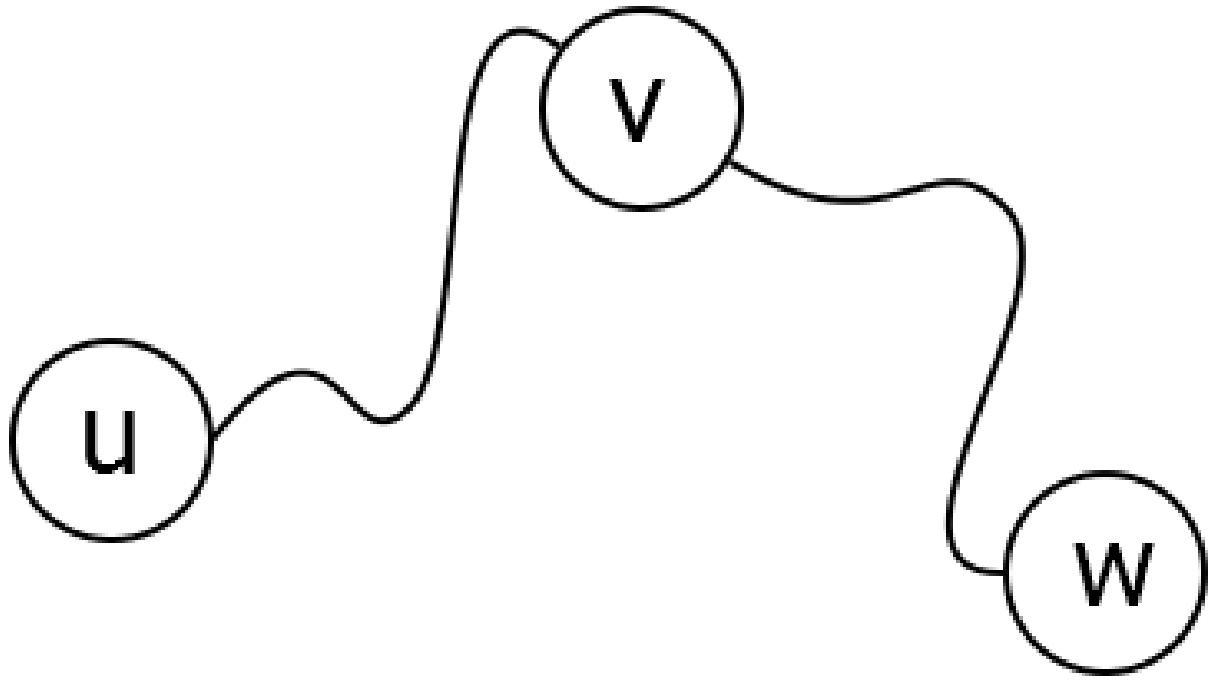
Proof

Proof.

Need to show reachability is an equivalence relation. Namely:

- v is reachable from v.
- If v reachable from w, w reachable from v.
- If v reachable from u, and w reachable from v, w reachable from u.

Proof (continued)



Problem

Connected Components

Input: Graph G

Output: The connected components of G

Outline

Idea

`Explore`(v) finds the connected component of v. Just need to repeat to find other components.

Idea

`Explore`(v) finds the connected component of v. Just need to repeat to find other components.
Modify DFS to do this.

Idea

`Explore`(v) finds the connected component of v. Just need to repeat to find other components.

Modify DFS to do this.

Modify goal to label connected components.

Modification of Explore

Explore(v)

```
visited(v) ← true
CCnum(v) ← cc
for (v, w) ∈ E:
    if not visited(w):
        Explore(w)
```

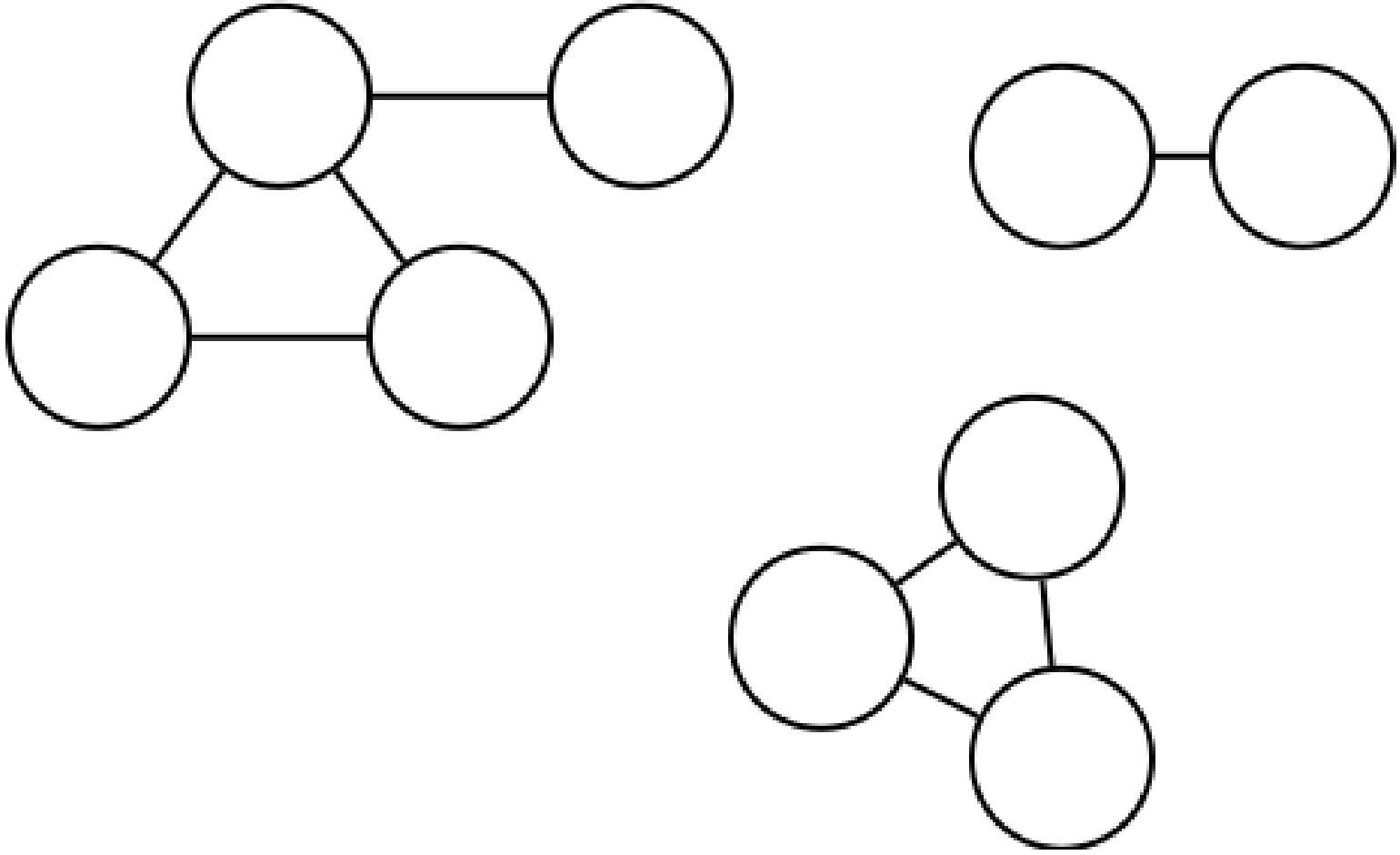
Modifications of DFS

DFS(G)

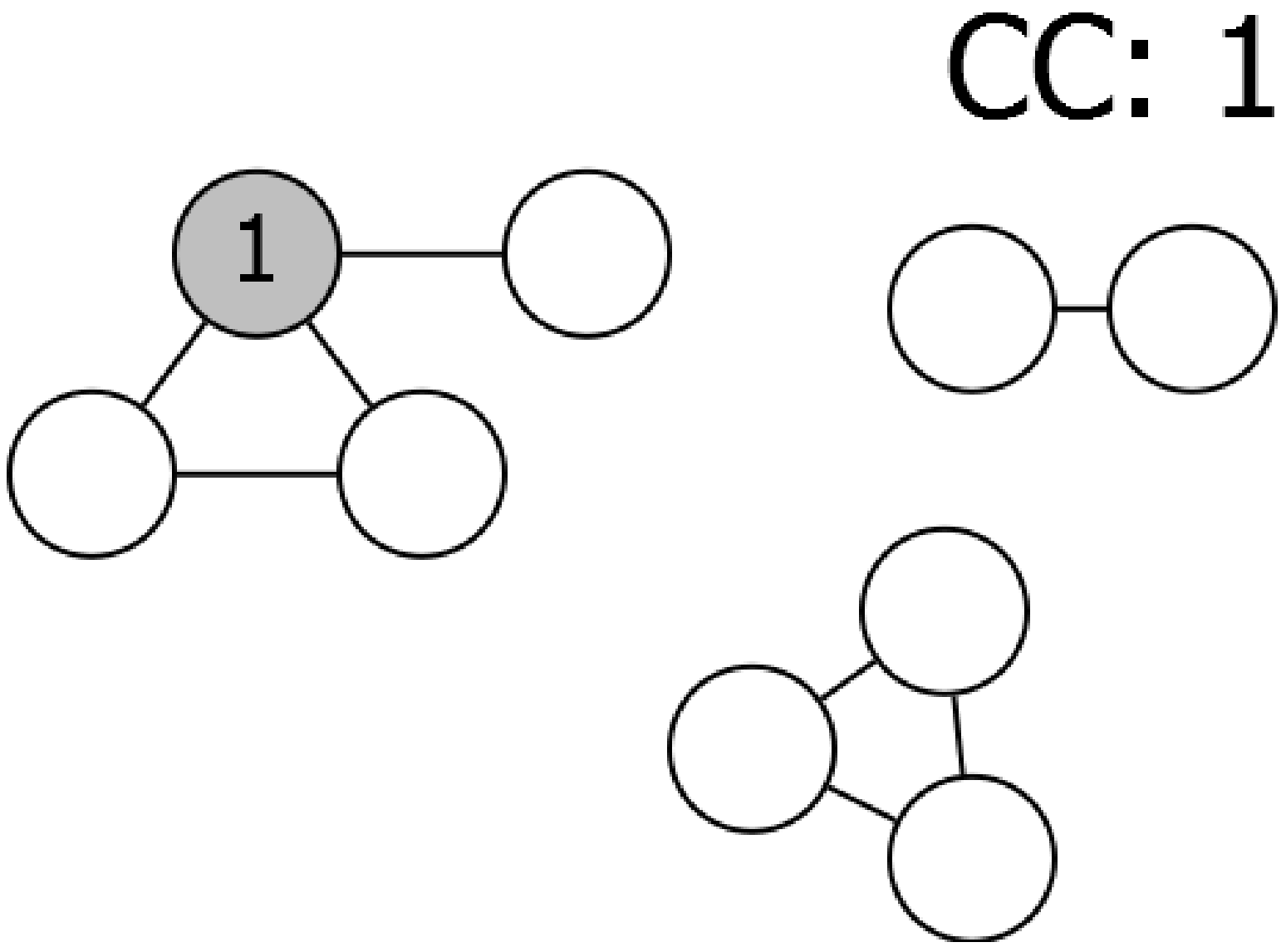
```
for all v ∈ V mark v unvisited
cc ← 1
for v ∈ V:
   if not visited(v):
      Explore(v)
      cc ← cc + 1
```

Example

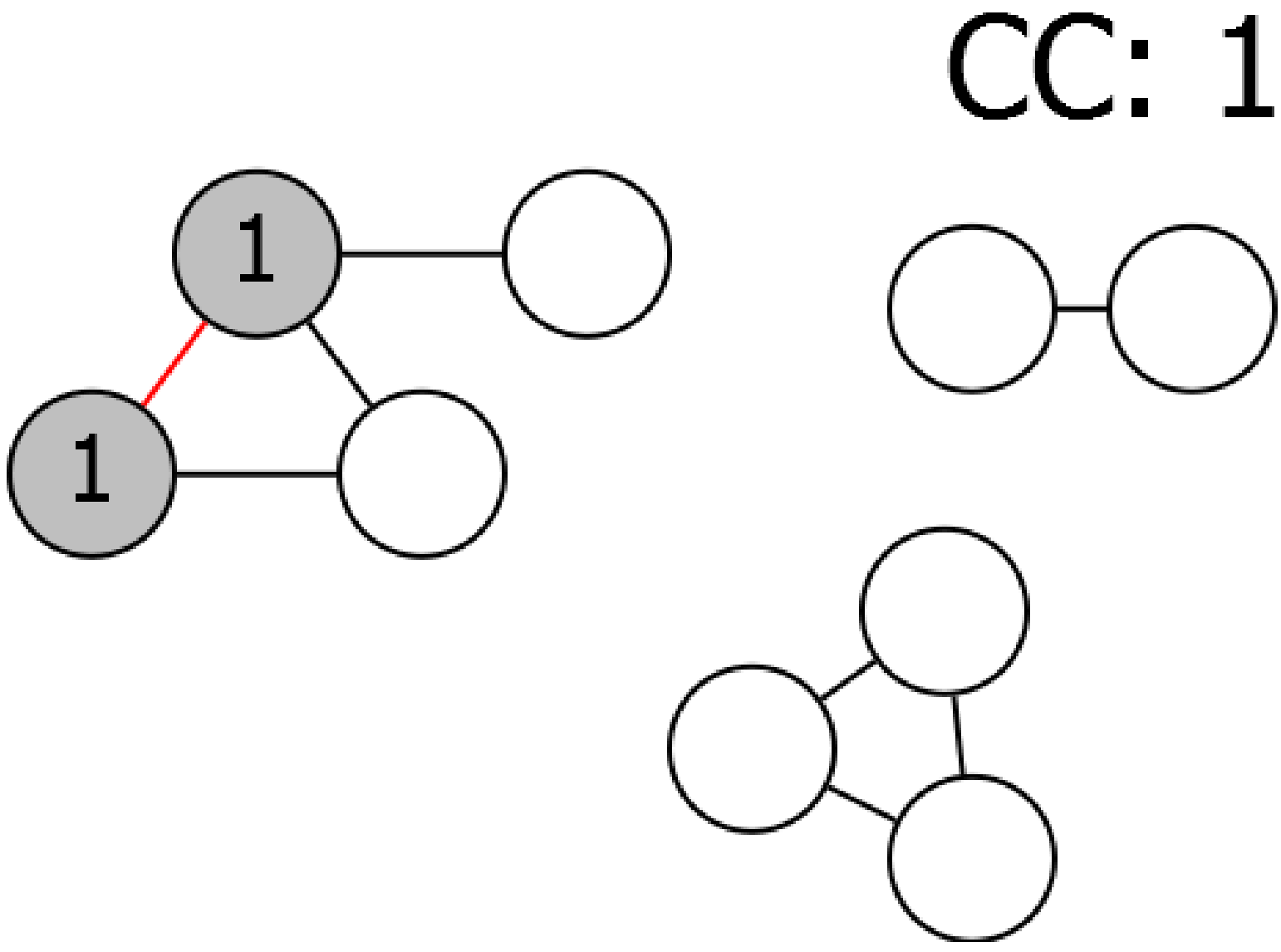
CC: 1



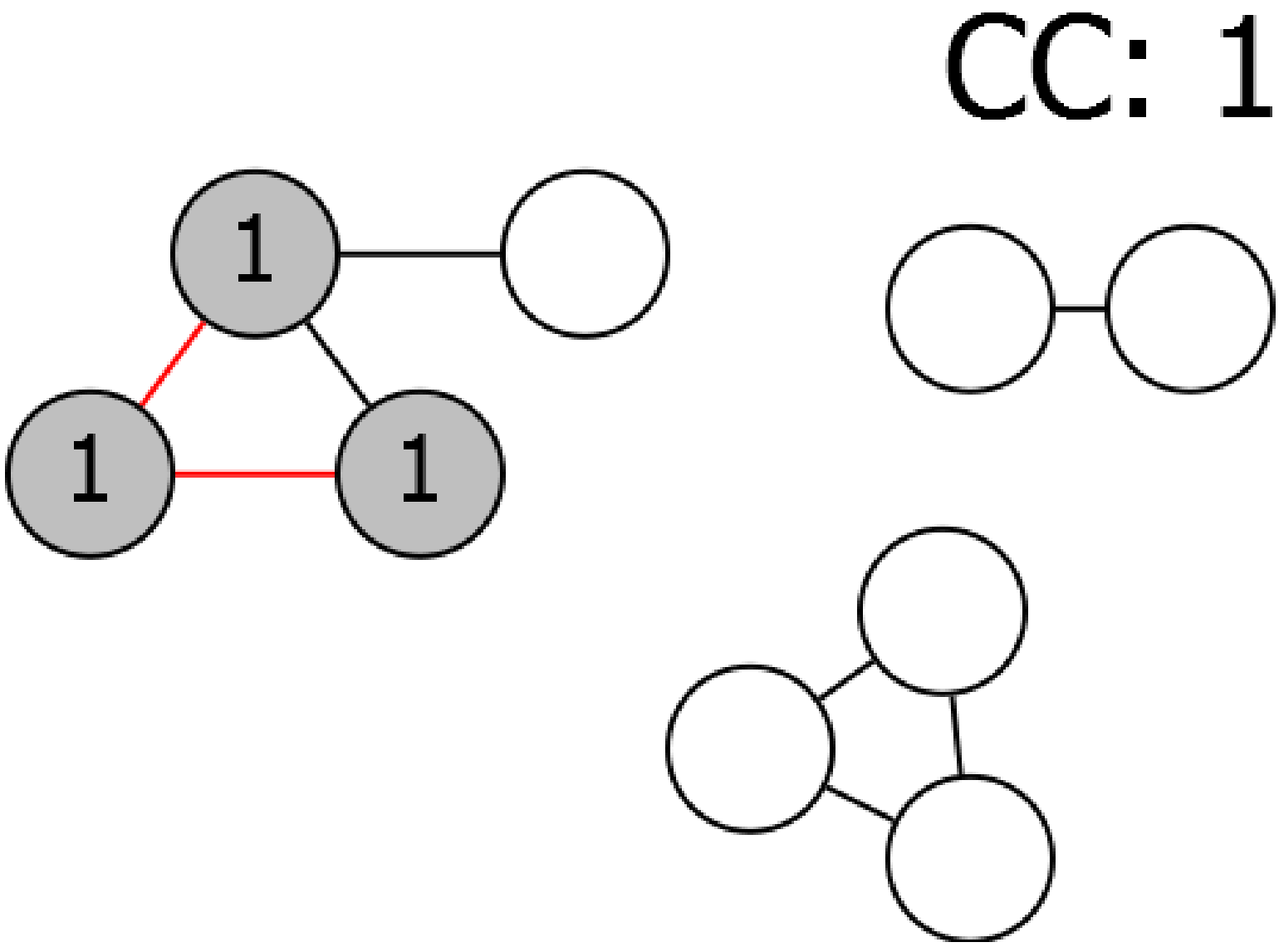
Example



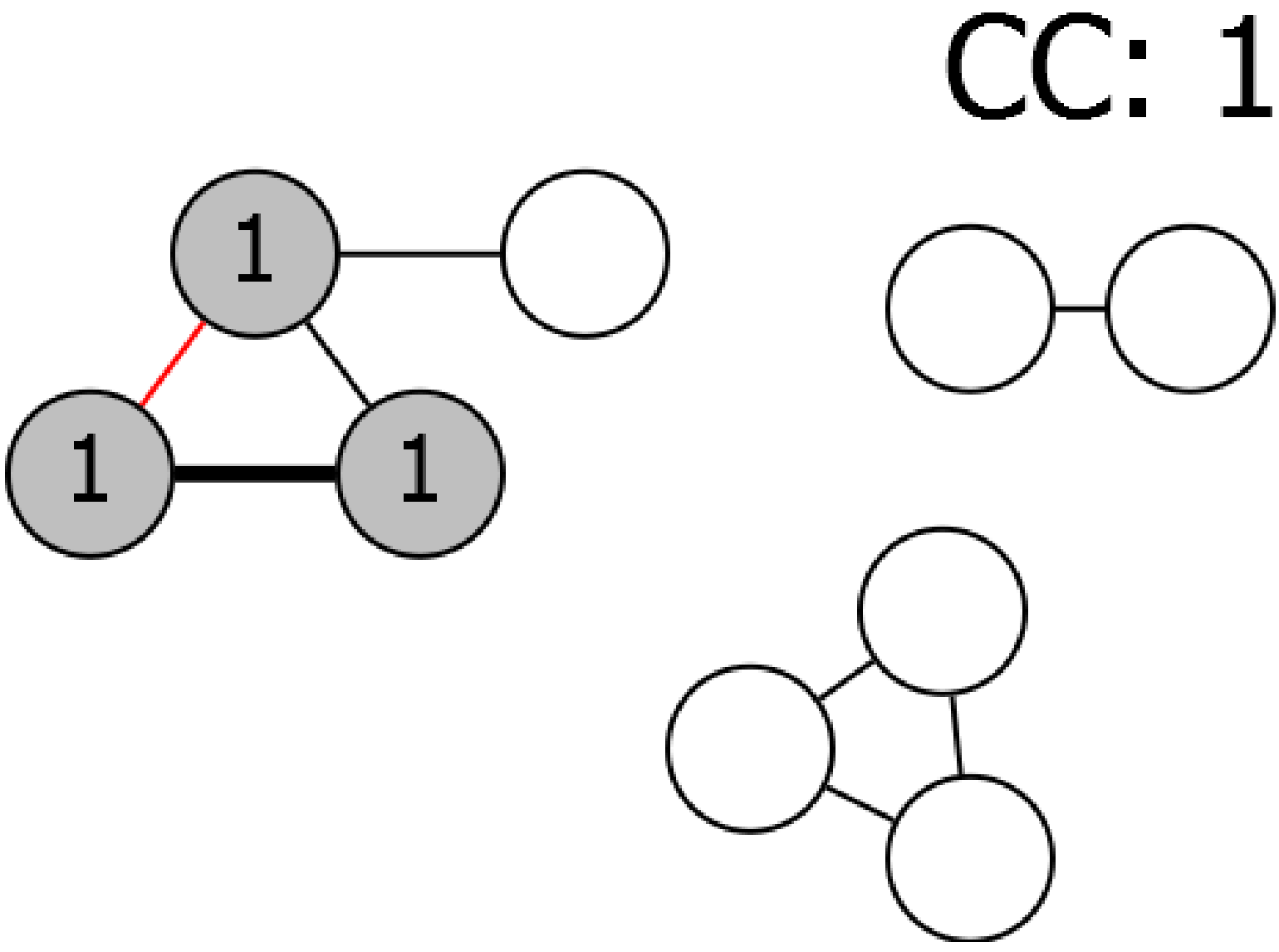
Example



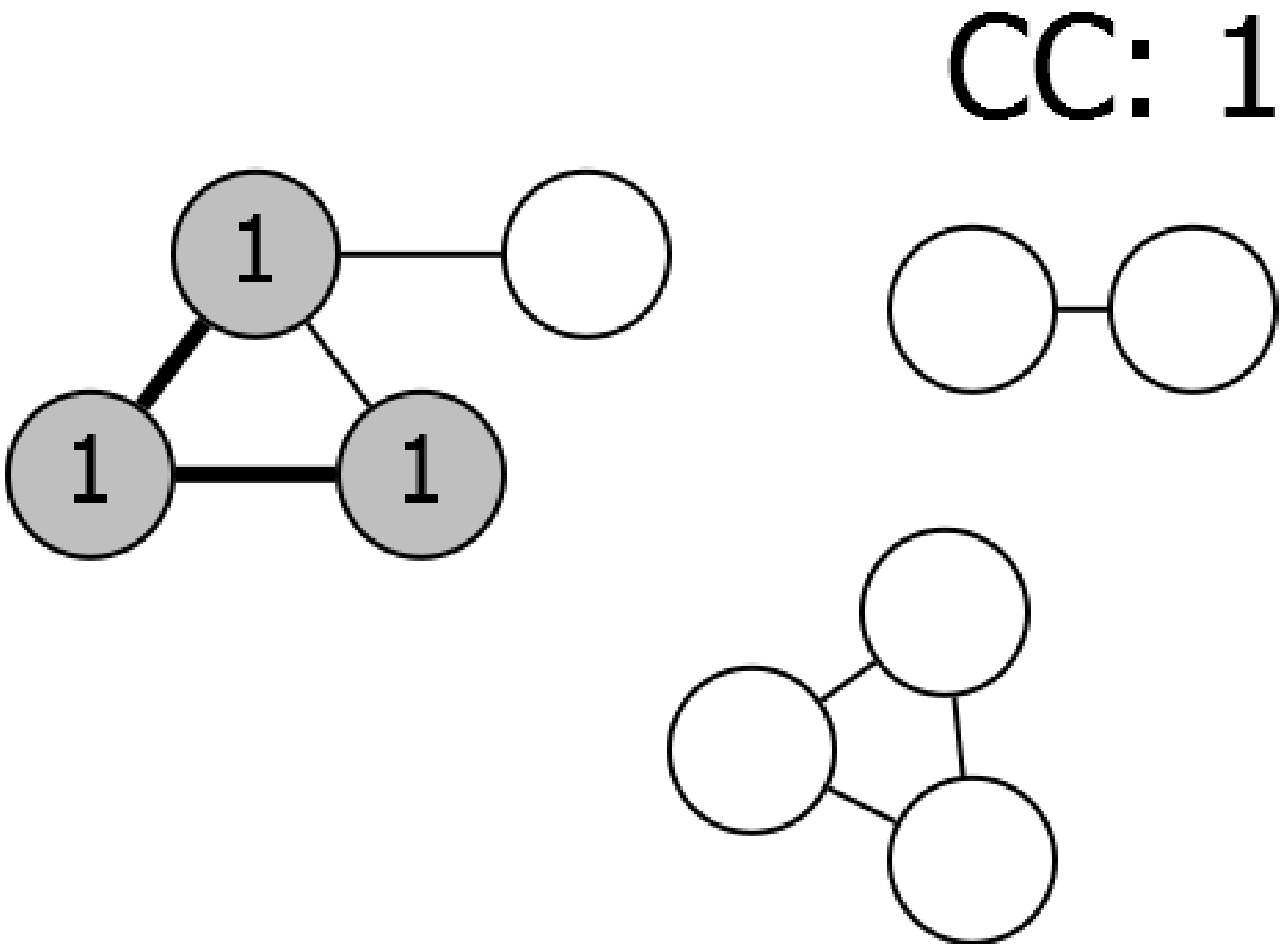
Example



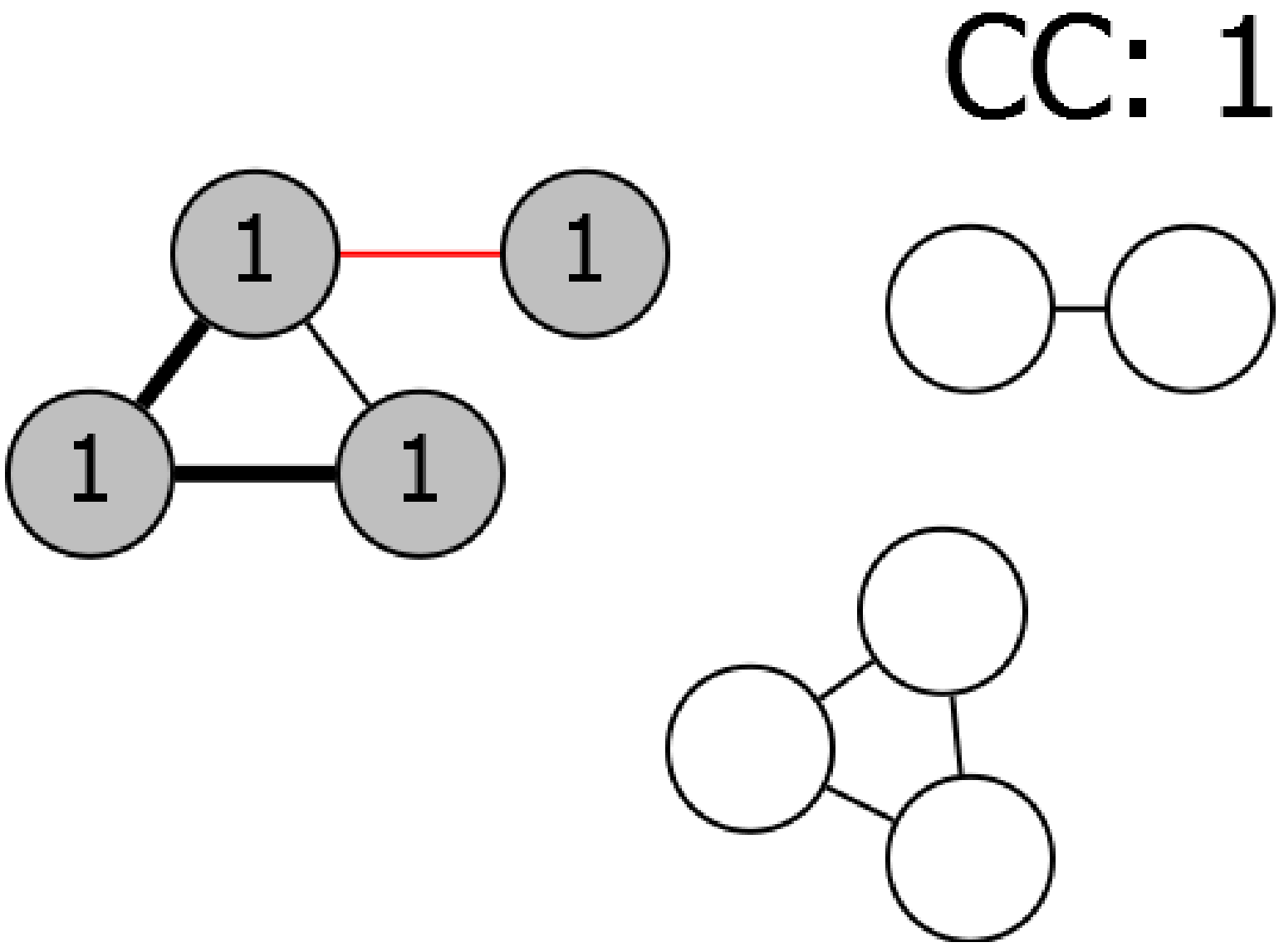
Example



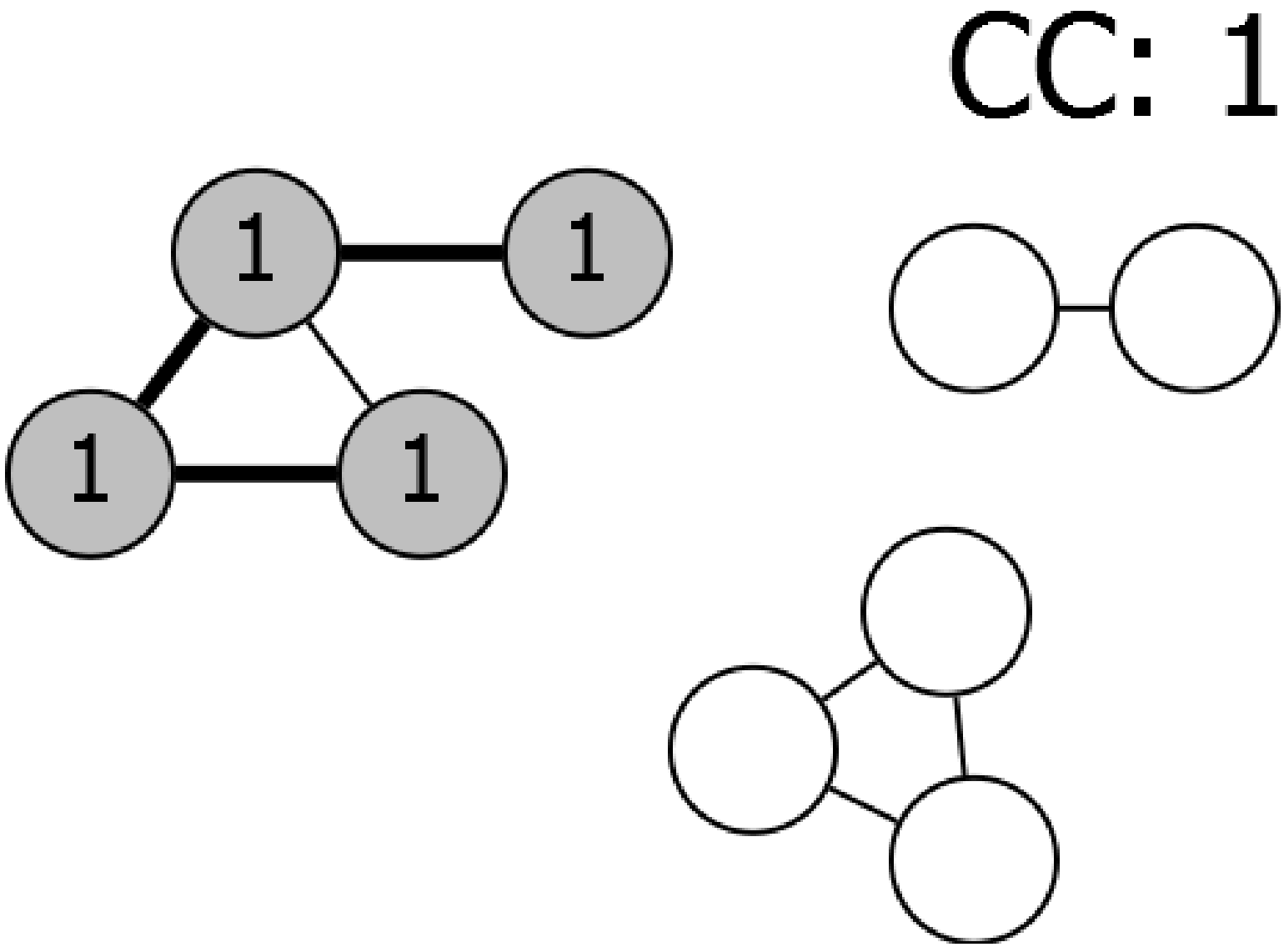
Example



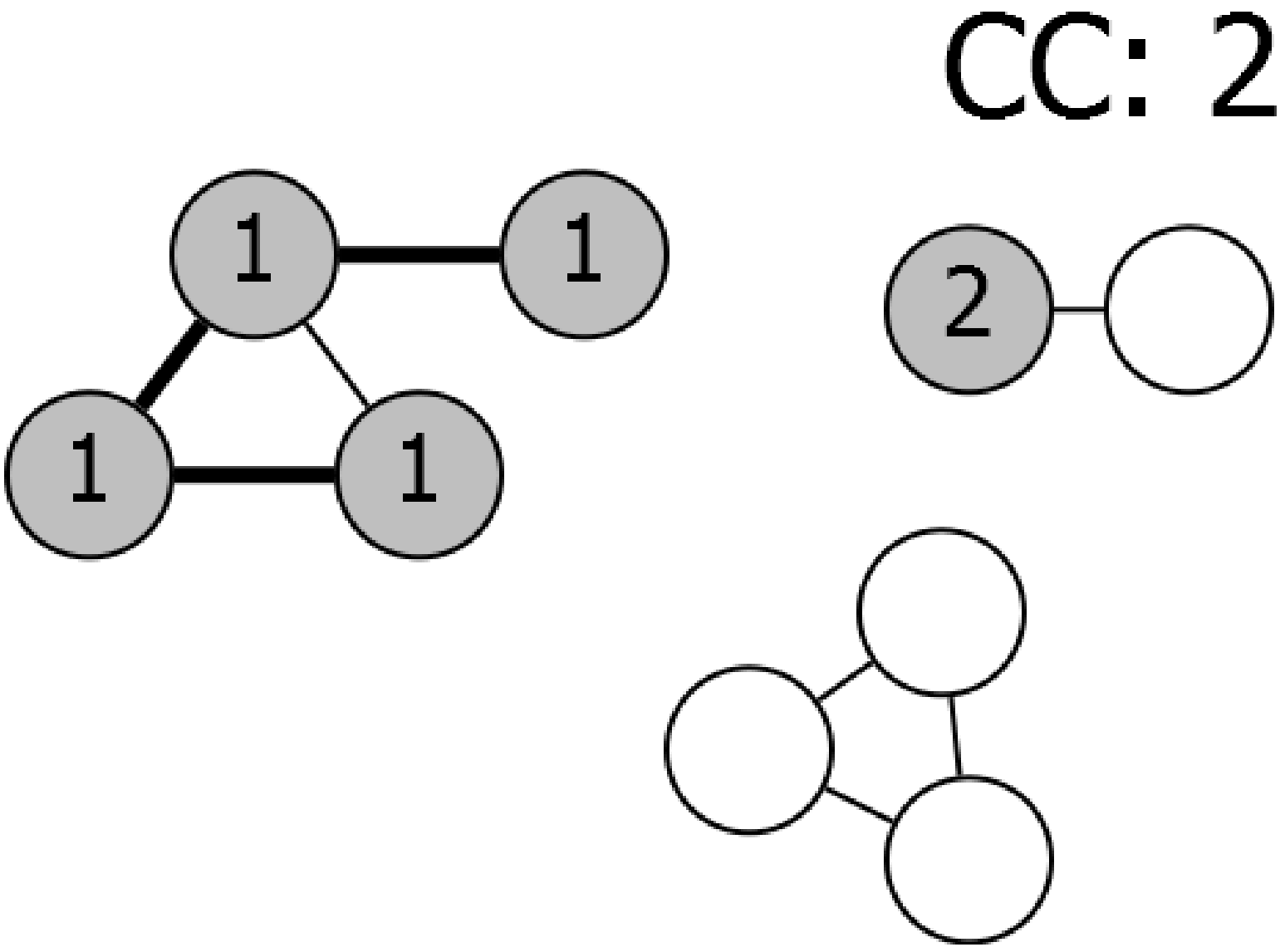
Example



Example

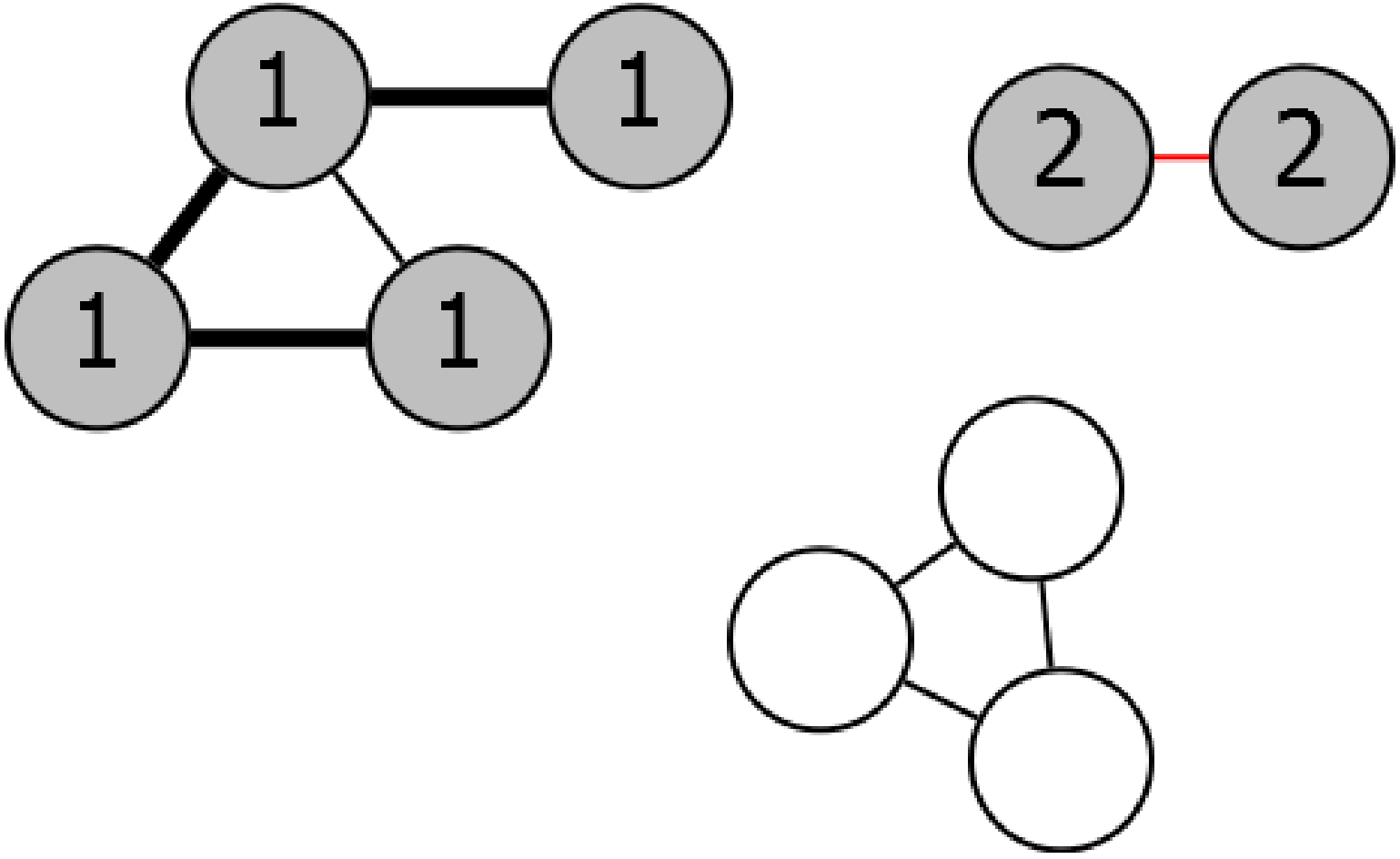


Example



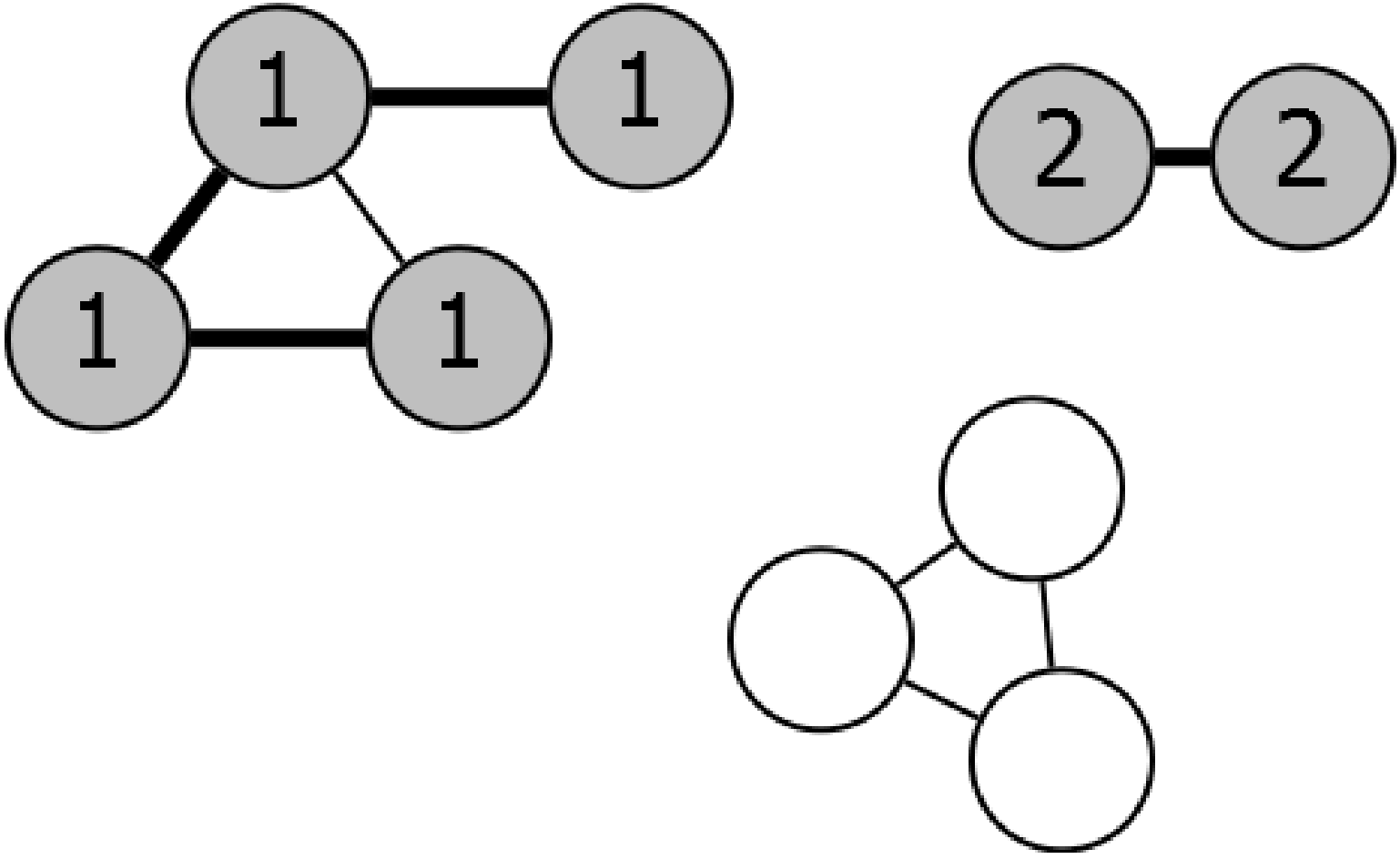
Example

CC: 2



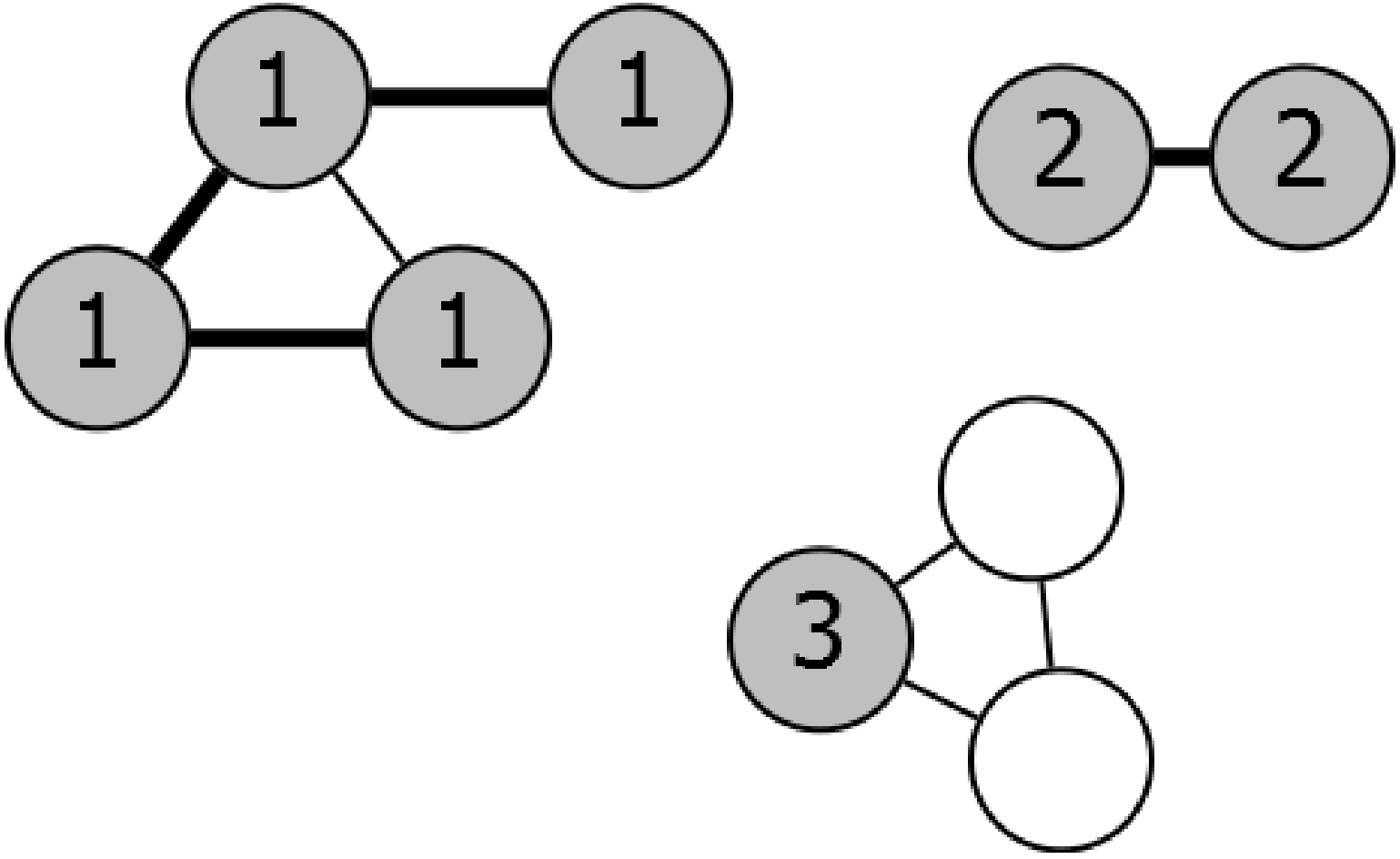
Example

CC: 2



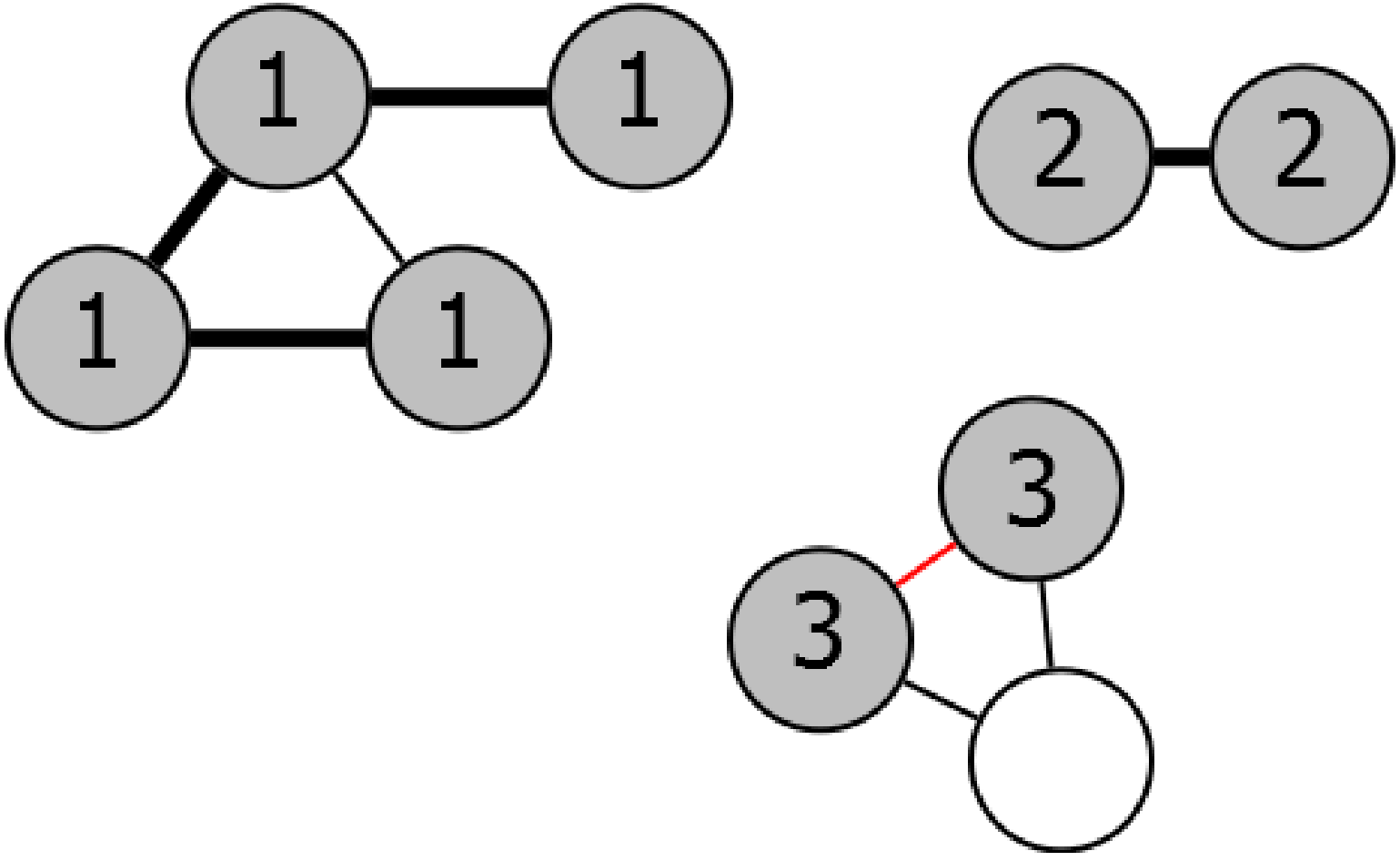
Example

CC: 3



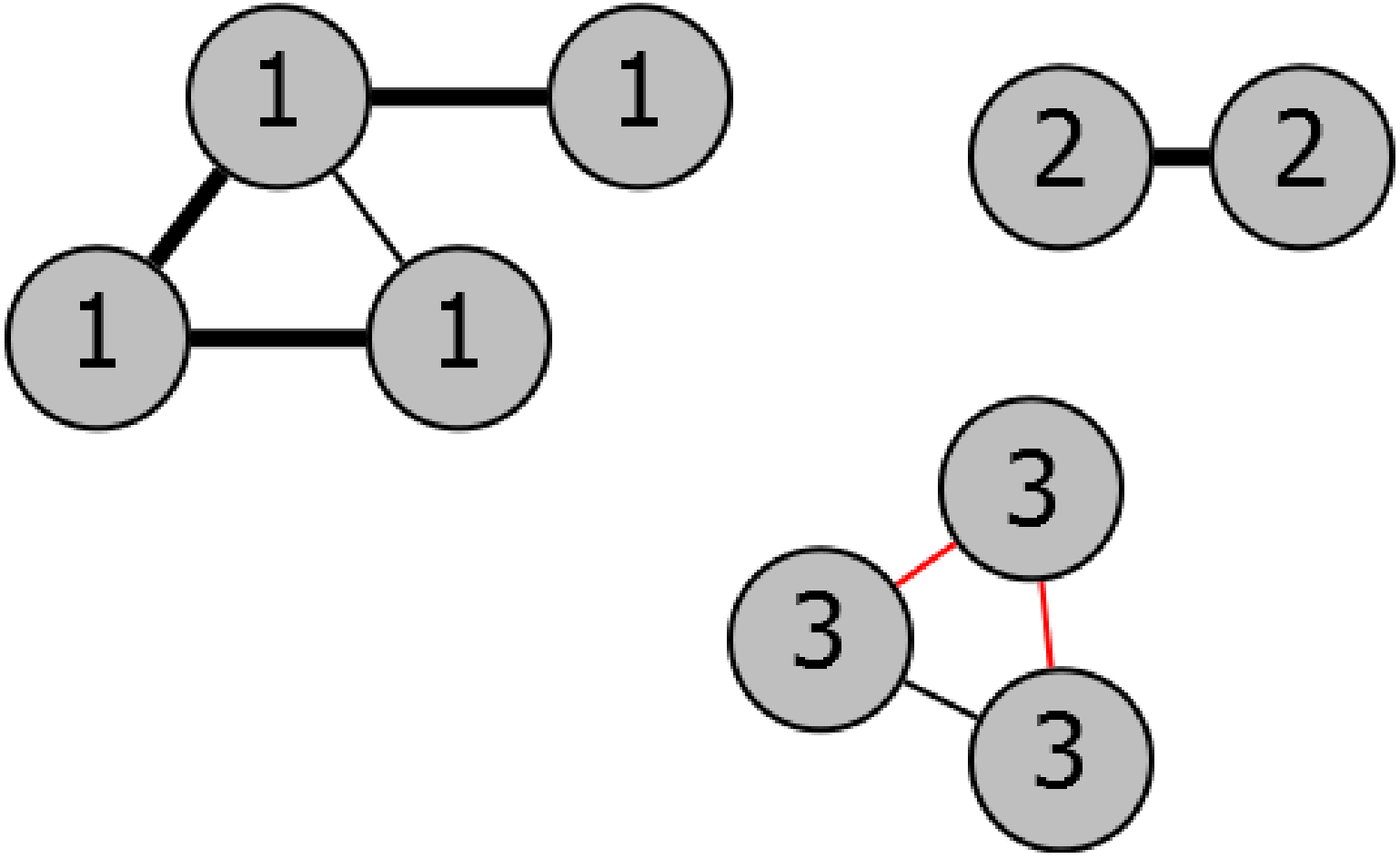
Example

CC: 3



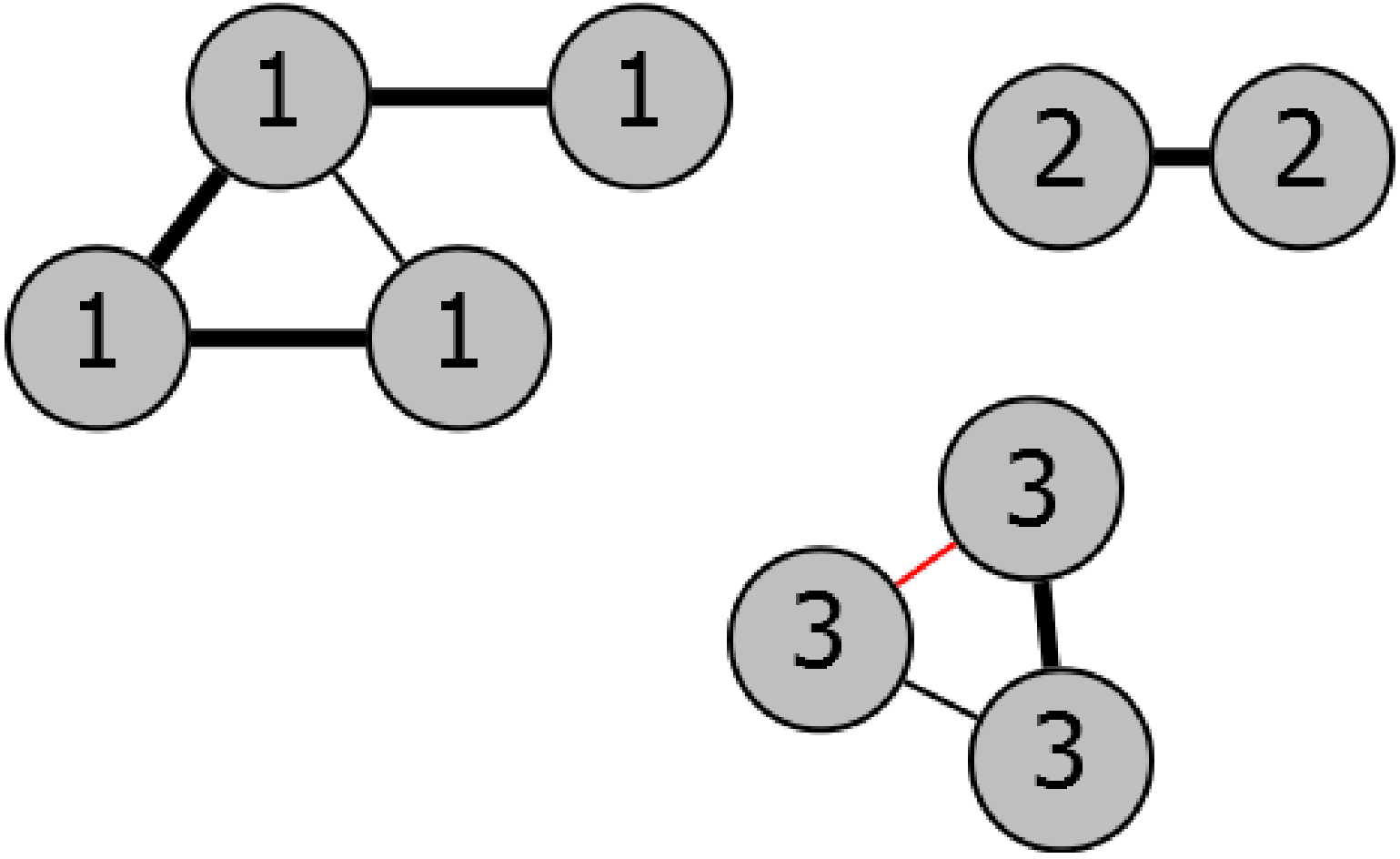
Example

CC: 3



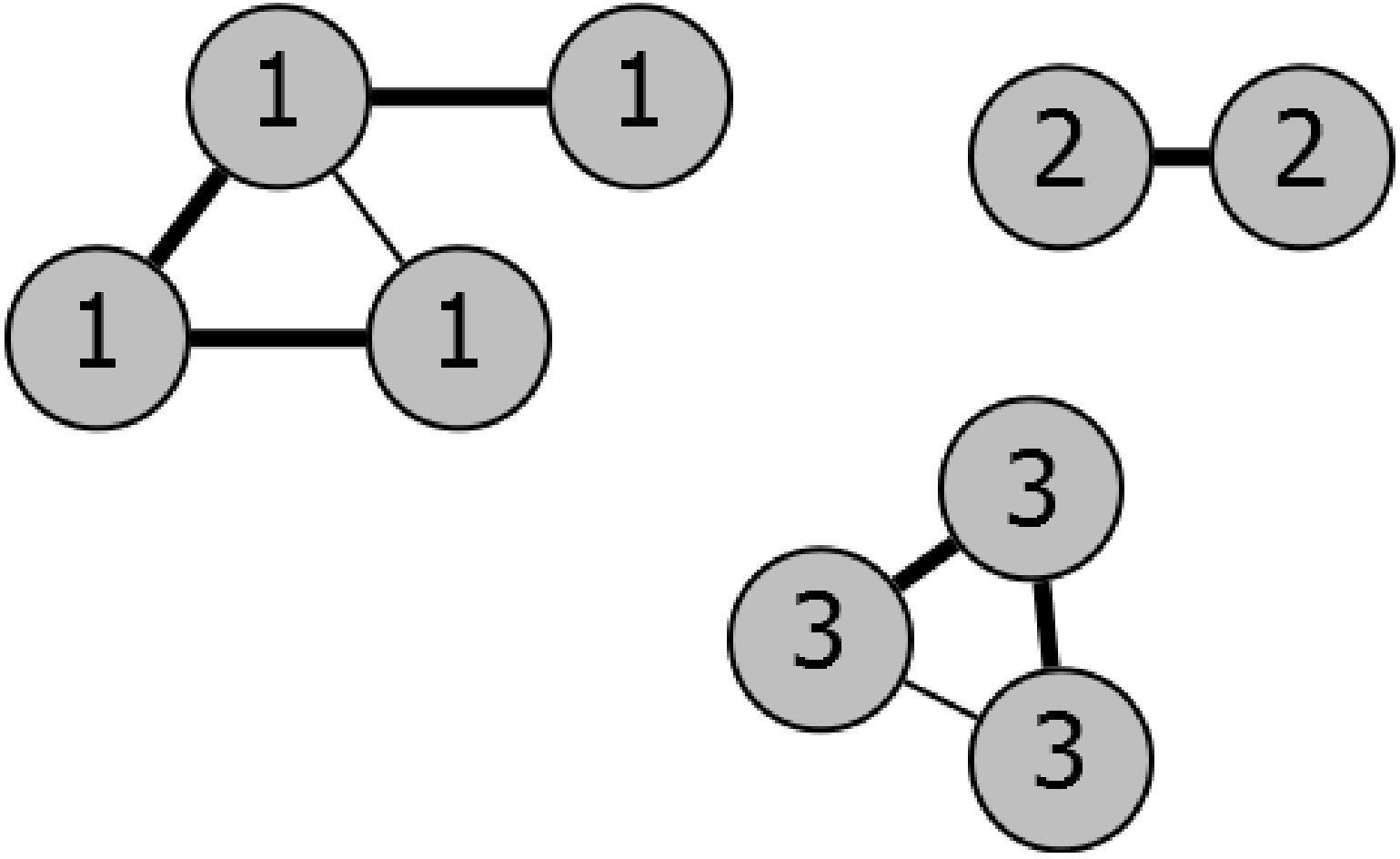
Example

CC: 3



Example

CC: 3



Correctness

- Each new `explore` finds new connected component.
- Eventually find every vertex.
- Runtime still $O(|V| + |E|)$.

Next Time

Other applications of `DFS`.