

Binary Search Trees: Applications

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Data Structures Fundamentals
Algorithms and Data Structures

Learning Objectives

- Compute order statistics in binary search trees.
- Use trees to store and manipulate sequential lists of elements.

Outline

1 Order Statistics

2 Color Flips

Problem

Things you might want to do:

- Return the 7th largest element.
- Return the median element.
- Return the 25% percentile element.

Order Statistics

Order Statistics

Input: The root of a tree T and a number k

Output: The k^{th} smallest element in T

Idea

- Need to know which subtree to look in.
- Need to know **how many** elements are in left subtree.

New Field

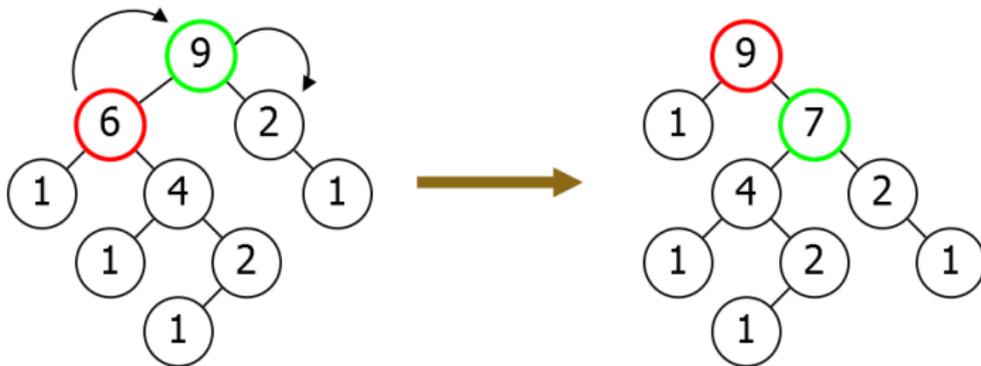
N .Size returns the number of elements in the subtree of N . Should satisfy:

$$N.Size = N.Left.Size + N.Right.Size + 1,$$

where null nodes have size zero.

Maintaining Value

When you rotate, you need to recompute sizes.



Recompute

RecomputeSize(N)

$N.Size \leftarrow N.Left.Size + N.Right.Size + 1$

Rotate

As before

RecomputeSize(Old root)

RecomputeSize(New root)

Order Statistics

`OrderStatistic(R, k)`

`$s \leftarrow R.Left.Size$`

`if $k = s + 1$:`

`return R`

`else if $k < s + 1$:`

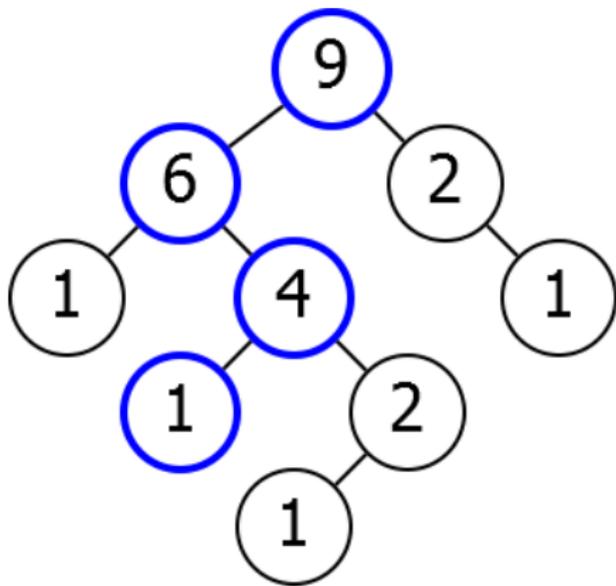
`return OrderStatistic($R.Left, k$)`

`else if $k > s + 1$:`

`return OrderStatistic($R.Right, k - s - 1$)`

Analysis

Runtime $O(h)$.



Puzzle

How do you compute the rank of the node with a given key?

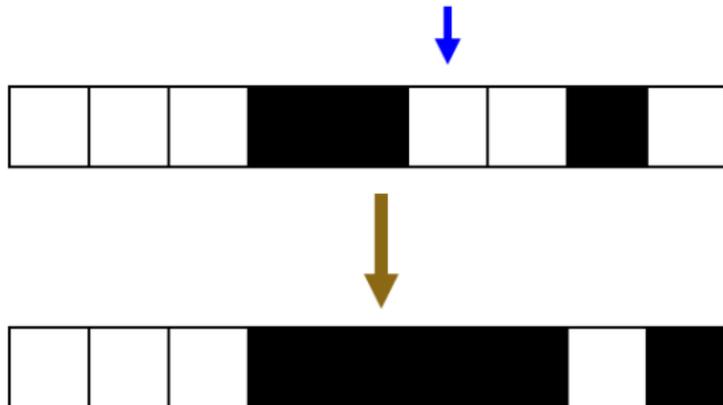
Outline

① Order Statistics

② Color Flips

Problem

- Array of squares.
- Each black or white.
- Want to be able to flip colors of all squares after index x .

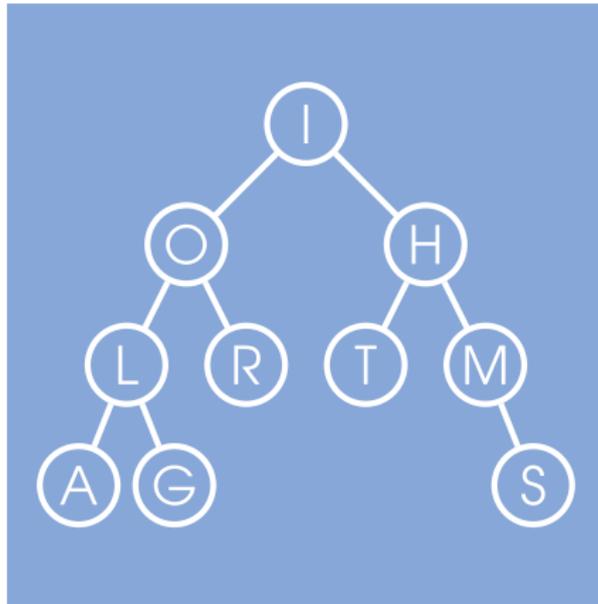


Operations

- `NewArray(n)` - Creates an array with n white squares.
- `Color(m)` - Returns color of m^{th} square.
- `Flip(x)` - Flips the color of all squares of index $> x$.

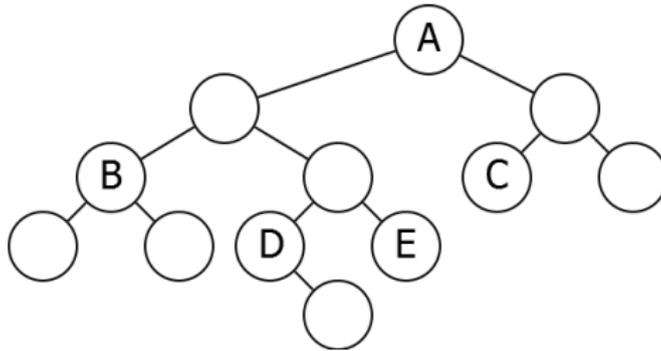
New Use For Trees

Store elements in sorted order.



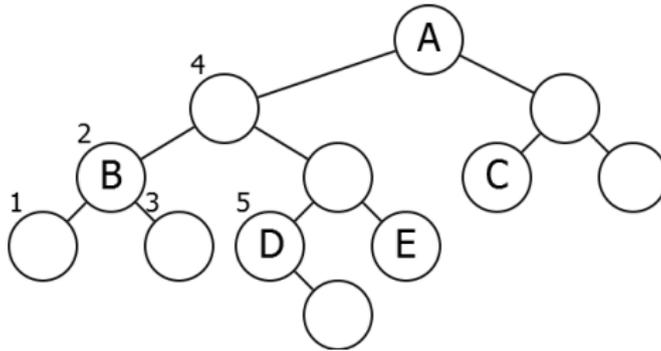
Problem

Which node represents the 5th smallest element in this tree?



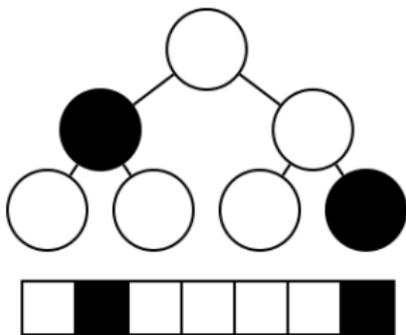
Problem

Which node represents the 5th smallest element in this tree?



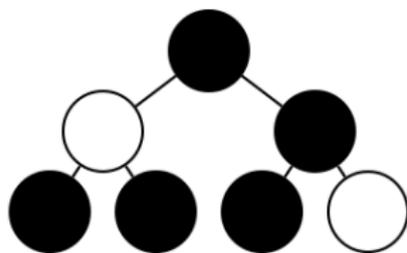
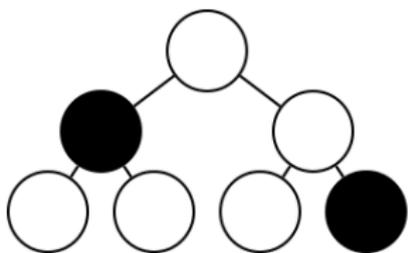
Idea

Store tree with nodes corresponding to list colors:



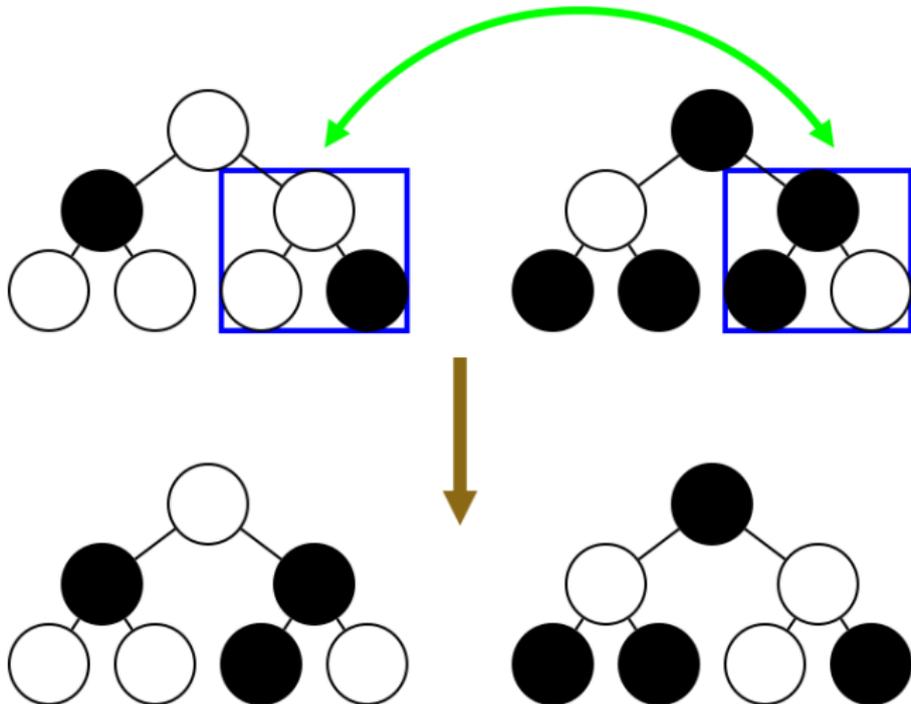
Idea II

Two trees- one with opposite colors:



Idea

Flip using merge and split



Create

`NewArray(n)`

Create two trees T_1, T_2 with keys $1 \dots n$.

Give nodes extra Color field.

All in T_1 have color White

All in T_2 have color Black

Find

`Color(m)`

`N ← Find(m, T1)`

`return N.Color`

Flip

Flip(x)

$(L_1, R_1) \leftarrow \text{Split}(T_1, x)$

$(L_2, R_2) \leftarrow \text{Split}(T_2, x)$

$\text{Merge}(L_1, R_2) \rightarrow T_1$

$\text{Merge}(L_2, R_1) \rightarrow T_2$

Moral

Trees can be used for more than searching.
Can be used to store lists.