Binary Search Trees: Introduction

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Data Structures Fundamentals
Algorithms and Data Structures

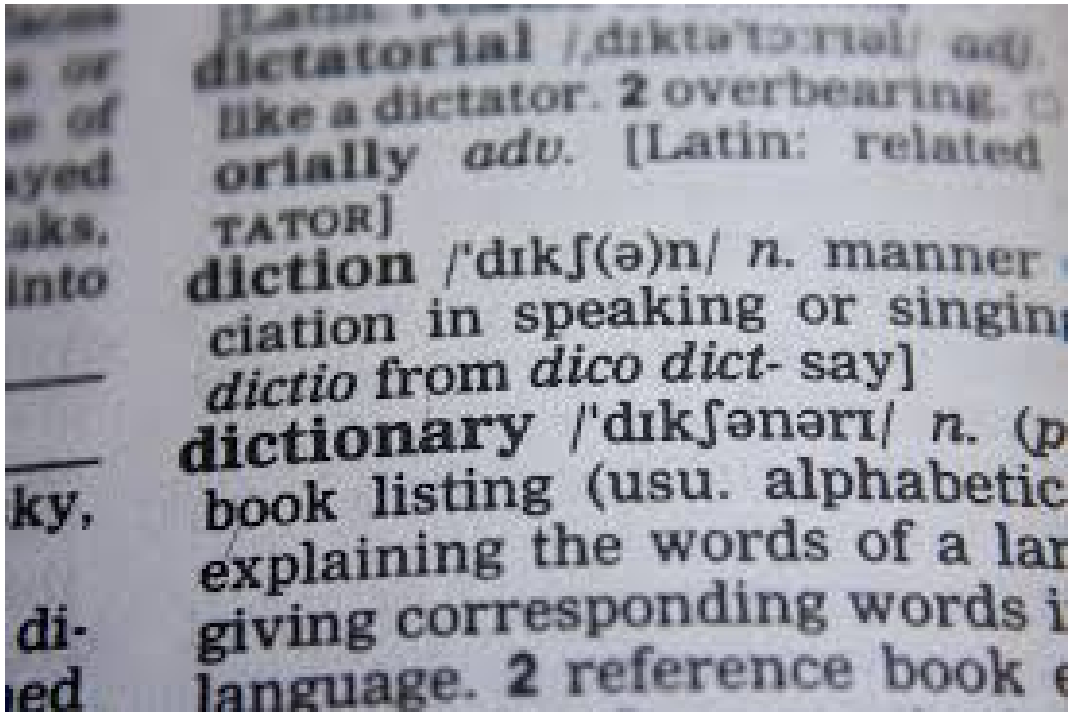
Learning Objectives

- Provide examples of the sorts of problems we hope to solve with Binary Search Trees.
- Show why data structures that we have already covered are insufficient.

Outline

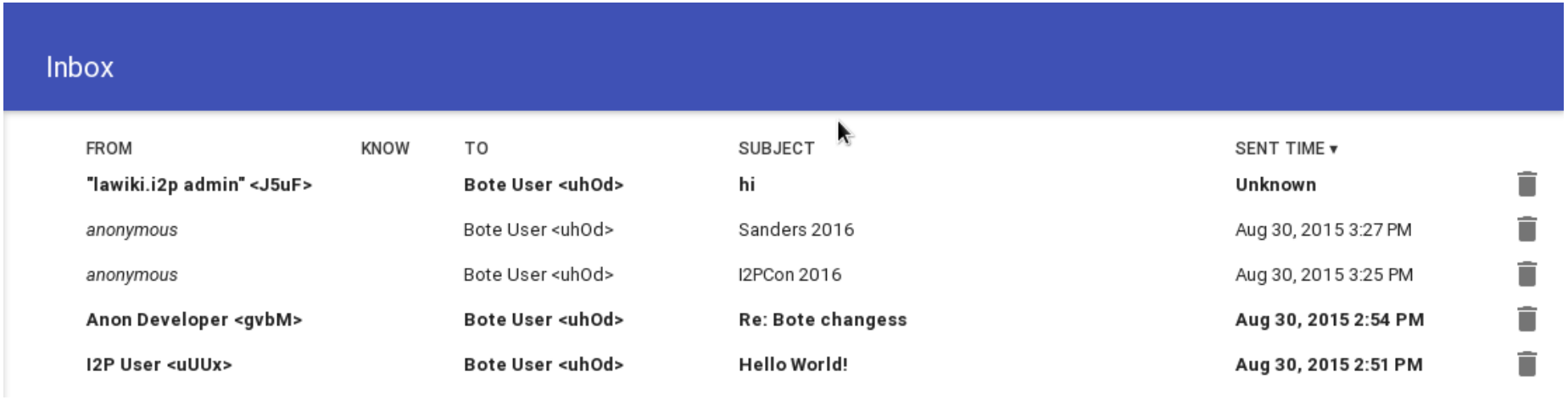
Dictionary Search

Find all words that start with some given string.



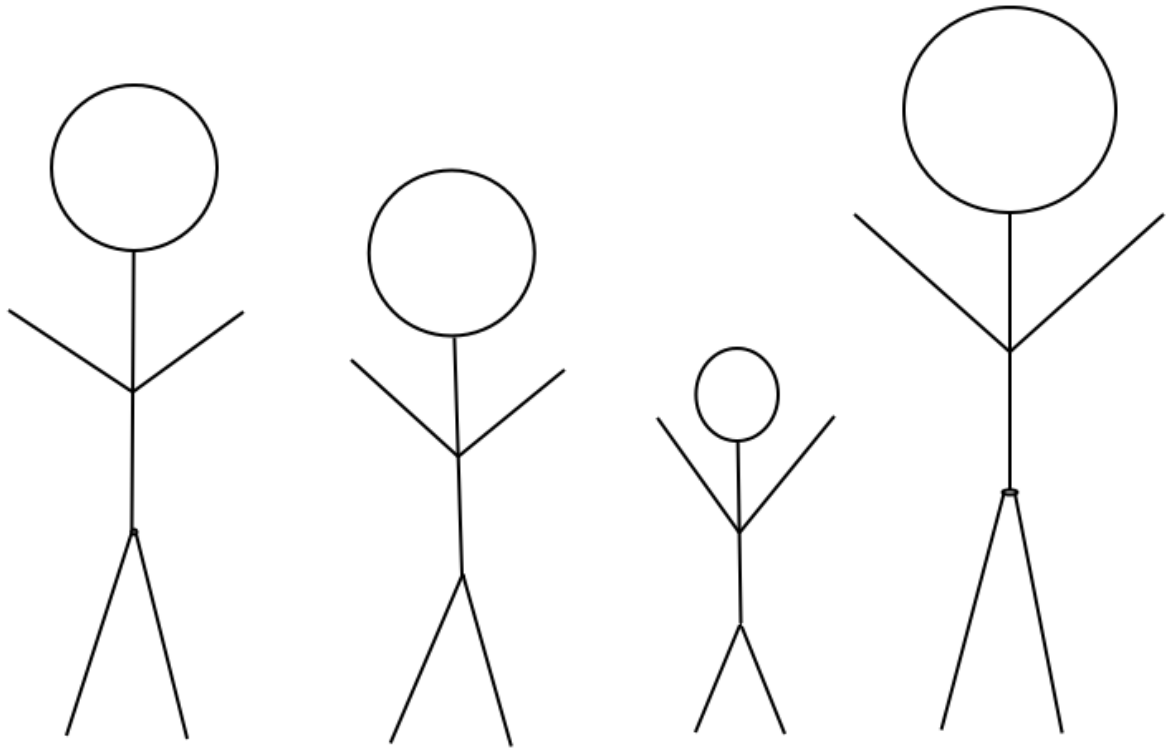
Date Ranges

Find all emails received in a given period.



Closest Height

Find the person in your class whose height is closest to yours.



Local Search

Definition

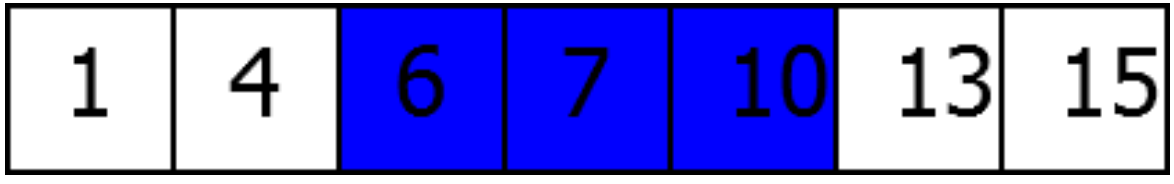
A Local Search Datastructure stores a number of elements each with a key coming from an ordered set. It supports operations:

- `RangeSearch`(x, y): Returns all elements with keys between x and y.
- `NearestNeighbors`(z): Returns the element with keys on either side of z.

Example

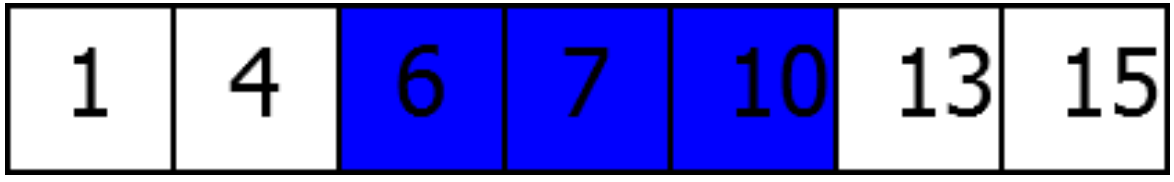
Example

`RangeSearch`$(5, 12)$

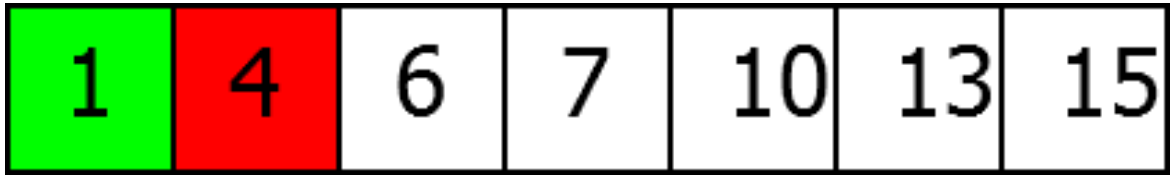


Example

`RangeSearch`$(5, 12)$



`NearestNeighbors`(3)



Dynamic Data Structure

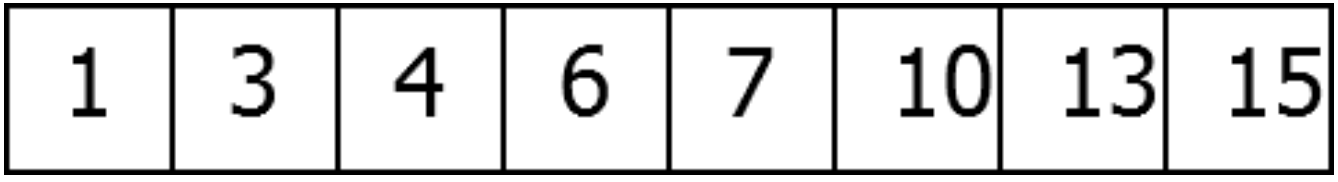
We would also like to be able to modify the data structure as we go.

- `Insert`(x): Adds a element with key x.
- `Delete`(x): Removes the element with key x.

Example

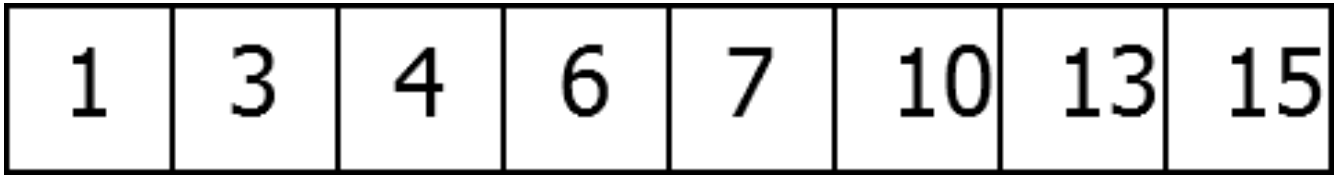
Example

`Insert(3)`



Example

`Insert(3)`



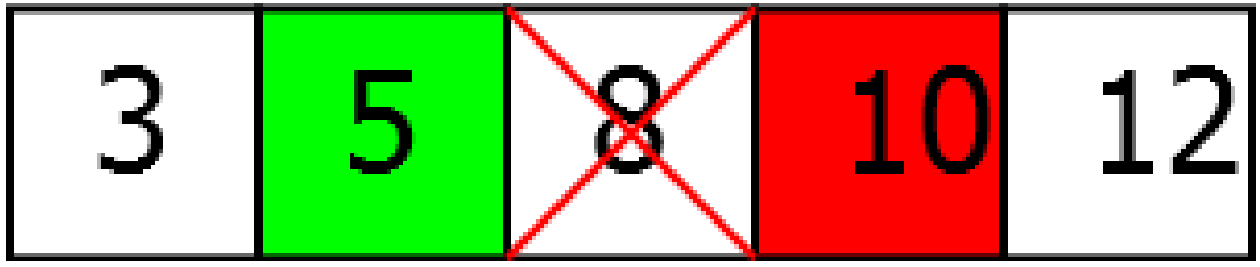
`Delete(10)`

Problem

If an empty data structure is given these commands what does it output at the end?

- `Insert(3)`
- `Insert(8)`
- `Insert(5)`
- `Insert(10)`
- `Delete(8)`
- `Insert(12)`
- `NearestNeighbors(7)`

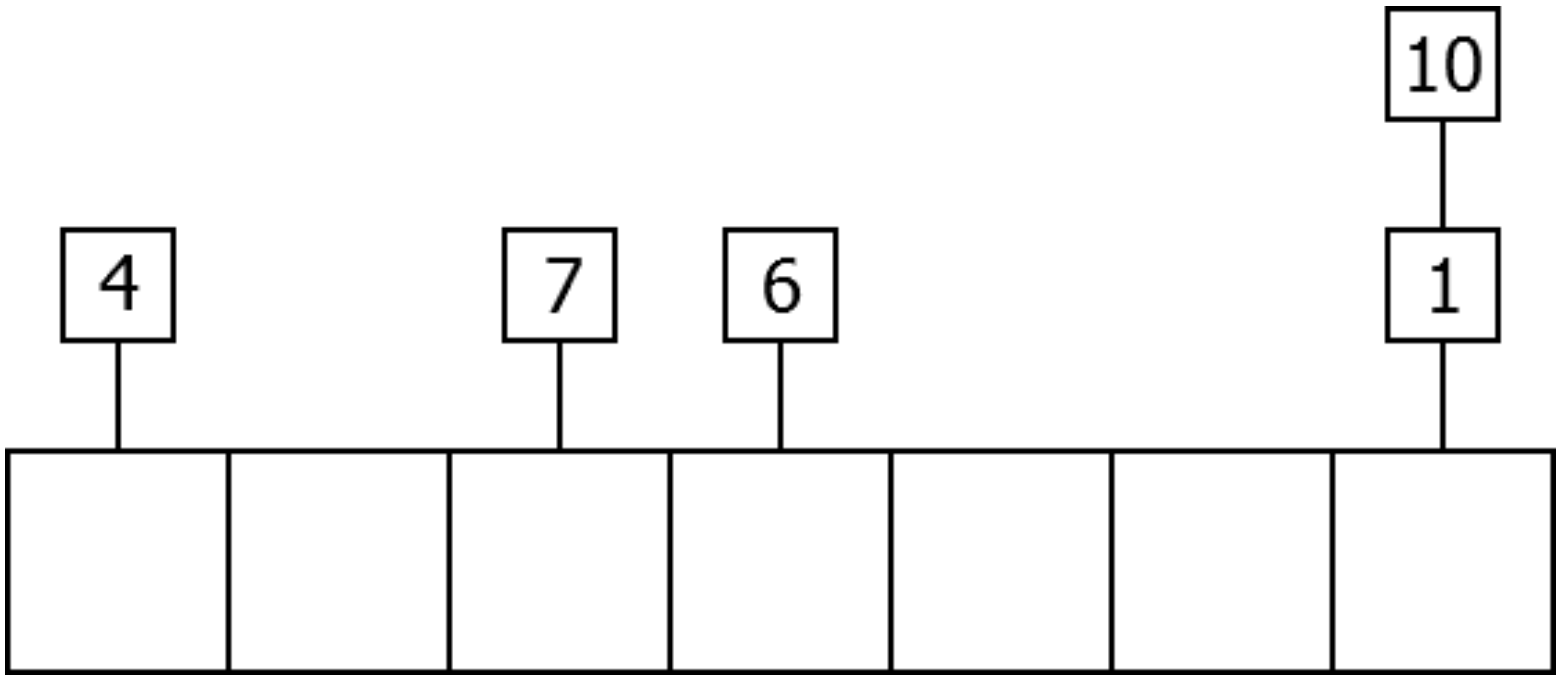
Answer



Outline

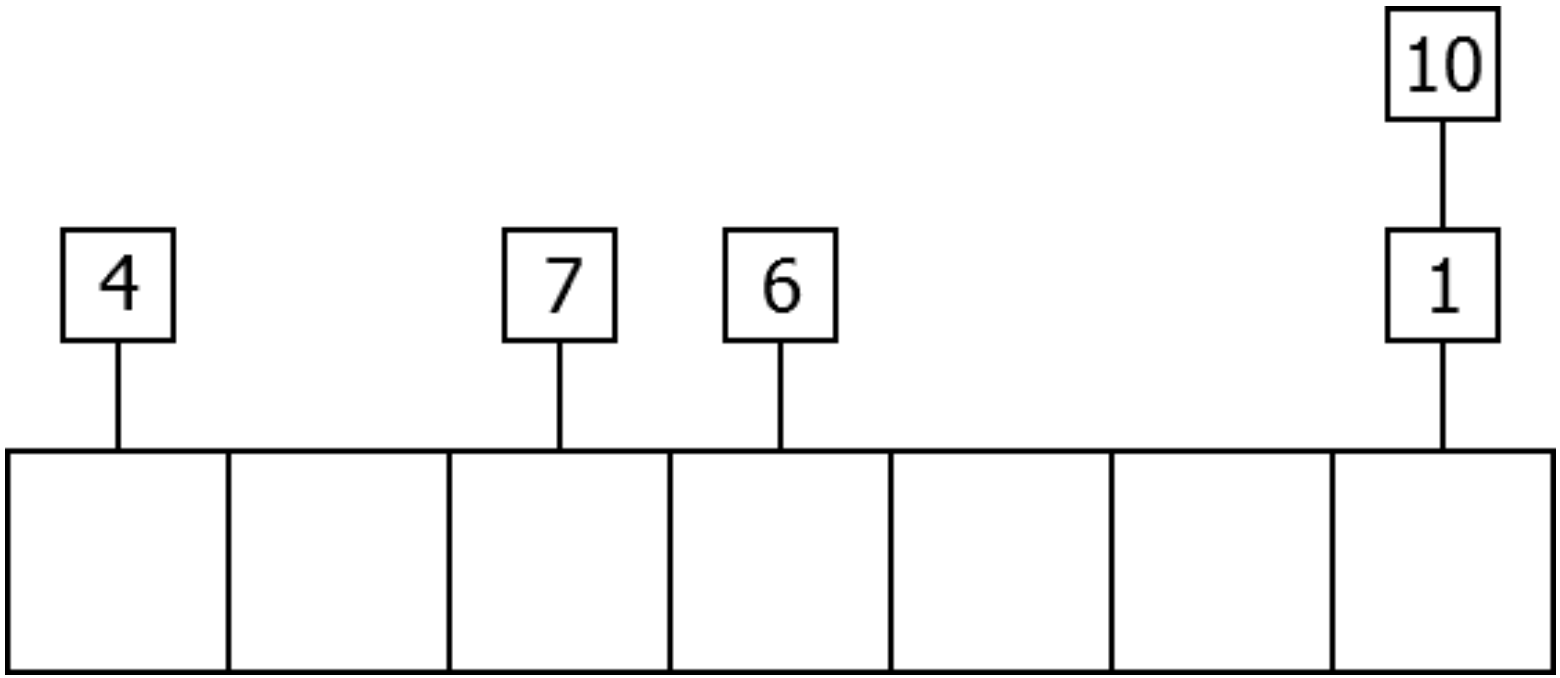
Hash Table

- `RangeSearch`: Impossible ×



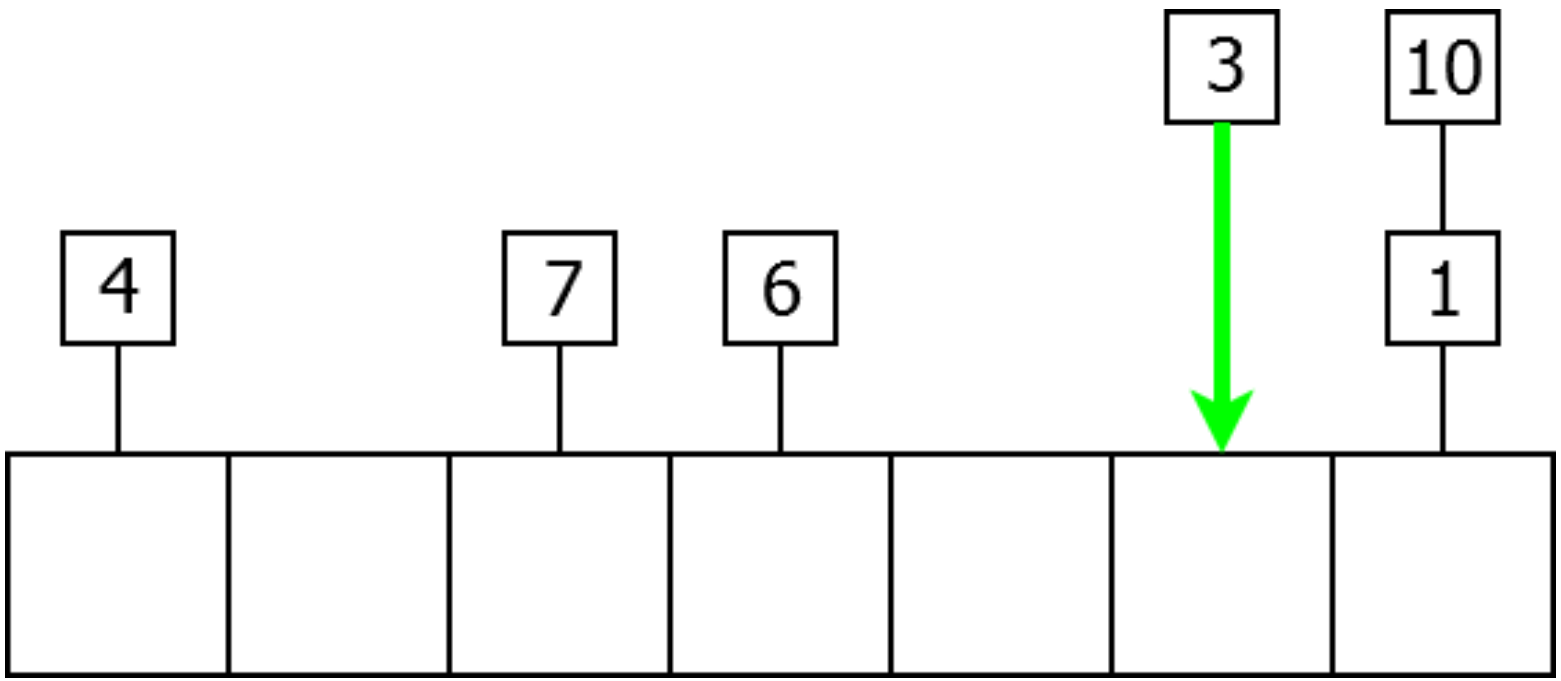
Hash Table

- `RangeSearch`: Impossible ×
- `NearestNeighbors`: Impossible ×



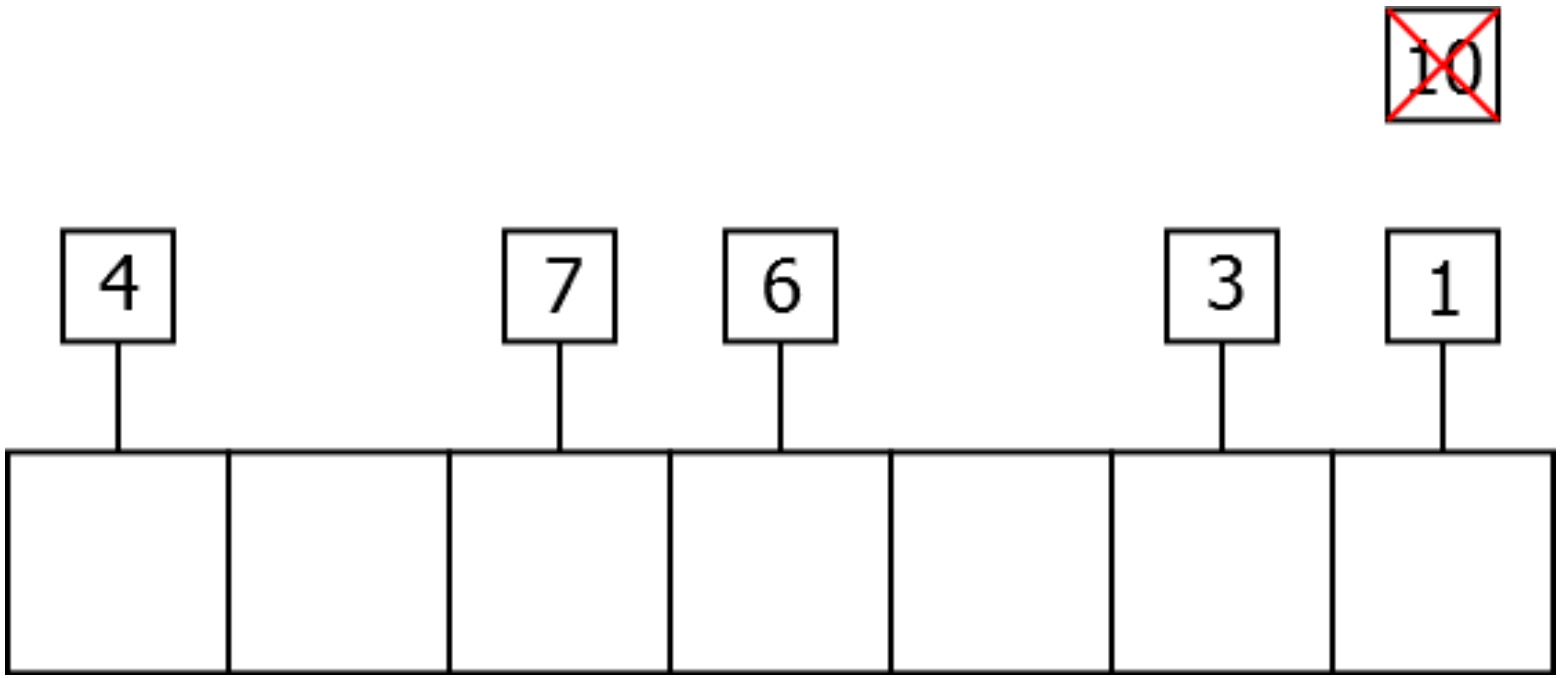
Hash Table

- `RangeSearch`: Impossible ×
- `NearestNeighbors`: Impossible ×
- `Insert`: $O(1)$ ✓



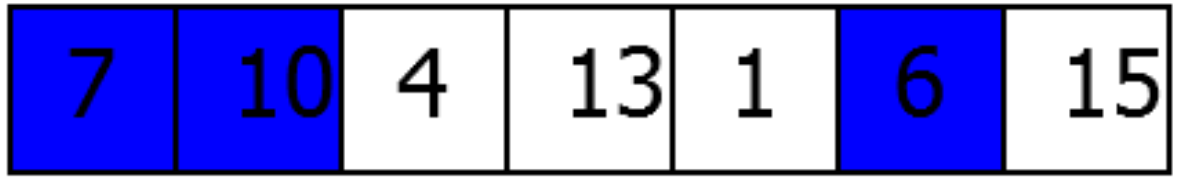
Hash Table

- `RangeSearch`: Impossible ×
- `NearestNeighbors`: Impossible ×
- `Insert`: $O(1)$ ✓
- `Delete`: $O(1)$ ✓



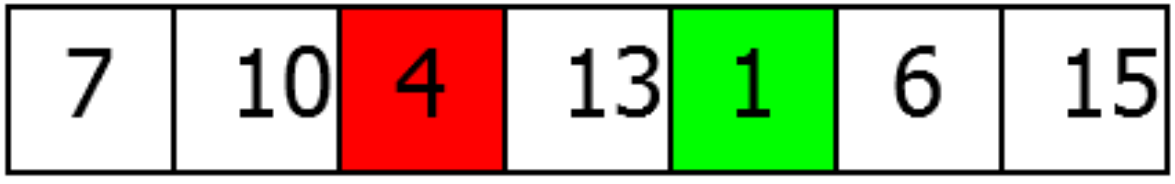
Array

- `RangeSearch`: $O(n)$ ×



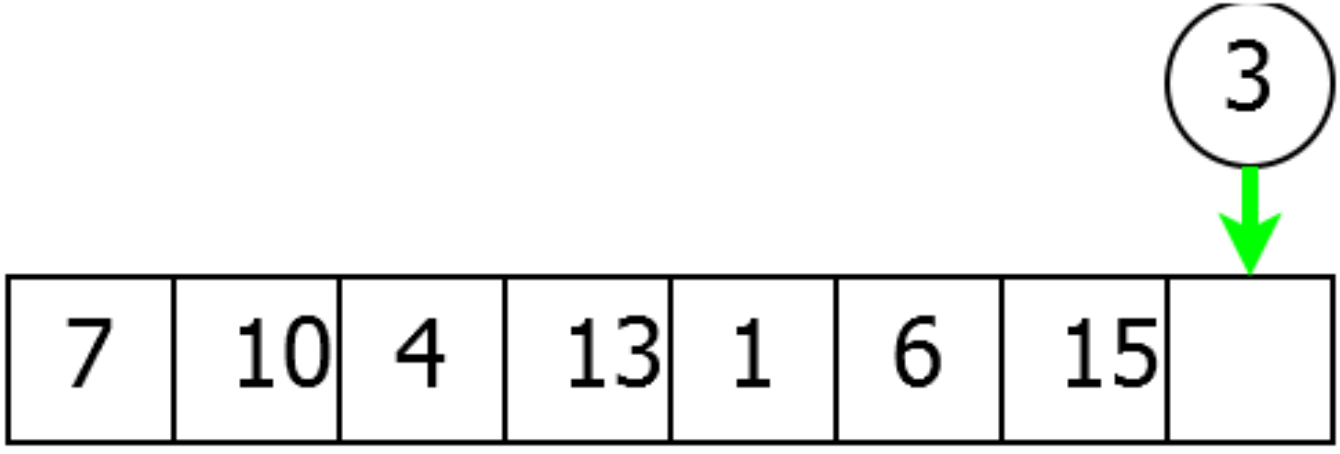
Array

- `RangeSearch`: $O(n)$ ×
- `NearestNeighbors`: $O(n)$ ×



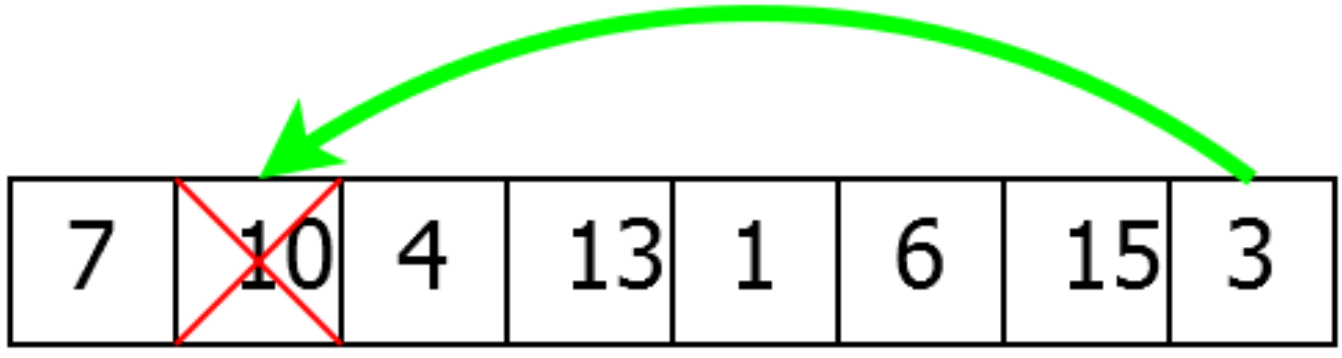
Array

- `RangeSearch`: $O(n)$ ✗
- `NearestNeighbors`: $O(n)$ ✗
- `Insert`: $O(1)$ ✓



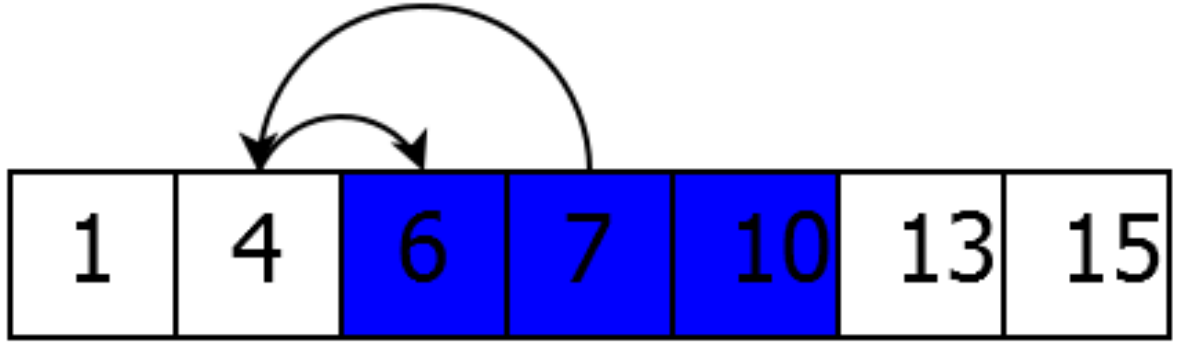
Array

- `RangeSearch`: $O(n)$ ✗
- `NearestNeighbors`: $O(n)$ ✗
- `Insert`: $O(1)$ ✓
- `Delete`: $O(1)$ ✓



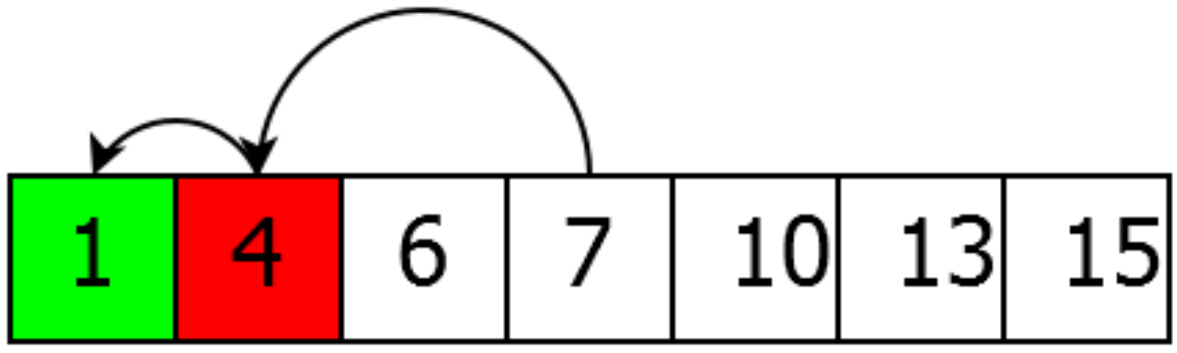
Sorted Array

- `RangeSearch`: $O(\log(n))$ ✓



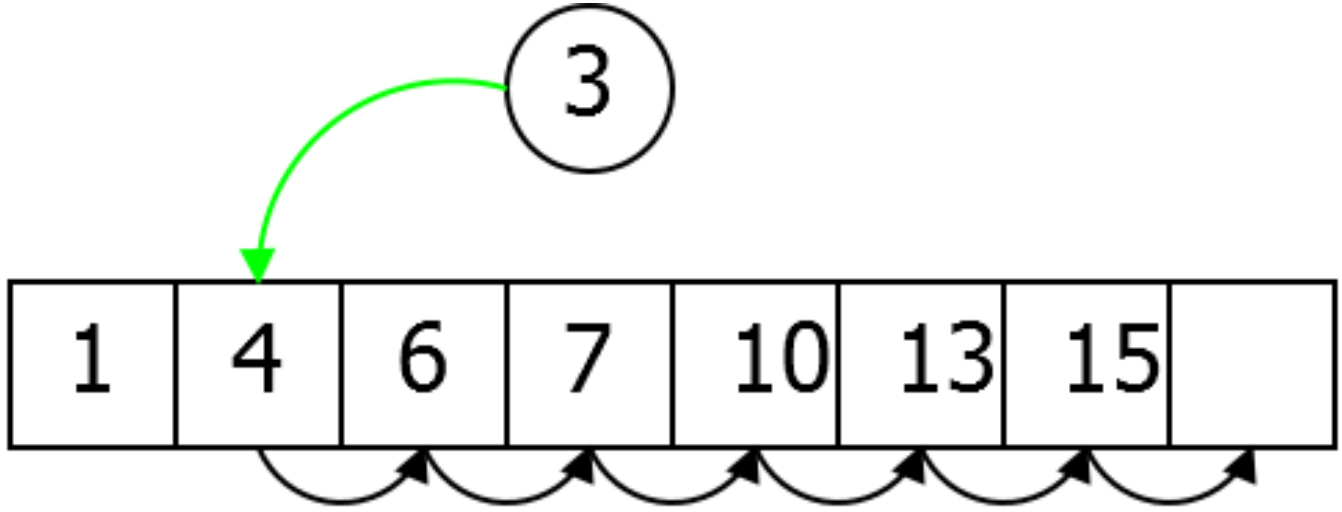
Sorted Array

- `RangeSearch`: $O(\log(n))$ ✓
- `NearestNeighbors`: $O(\log(n))$ ✓



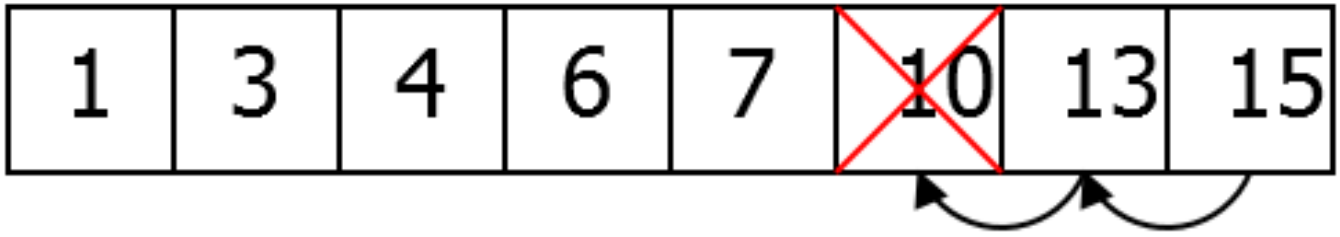
Sorted Array

- `RangeSearch`: $O(\log(n))$ ✓
- `NearestNeighbors`: $O(\log(n))$ ✓
- `Insert`: $O(n)$ ×



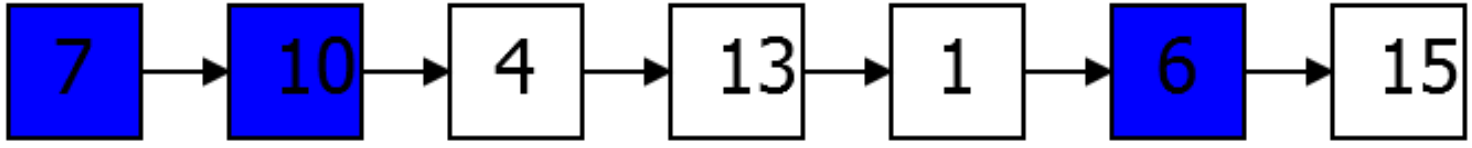
Sorted Array

- `RangeSearch`: $O(\log(n))$ ✓
- `NearestNeighbors`: $O(\log(n))$ ✓
- `Insert`: $O(n)$ ×
- `Delete`: $O(n)$ ×



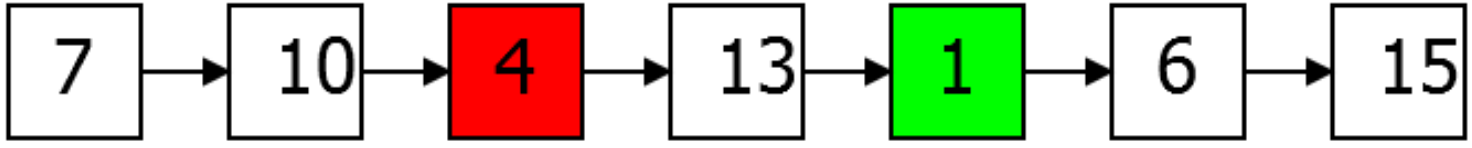
Linked List

- `RangeSearch`: $O(n)$ ×



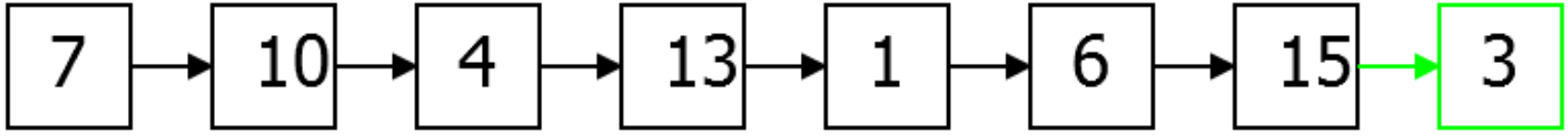
Linked List

- `RangeSearch`: $O(n)$ ×
- `NearestNeighbors`: $O(n)$ ×



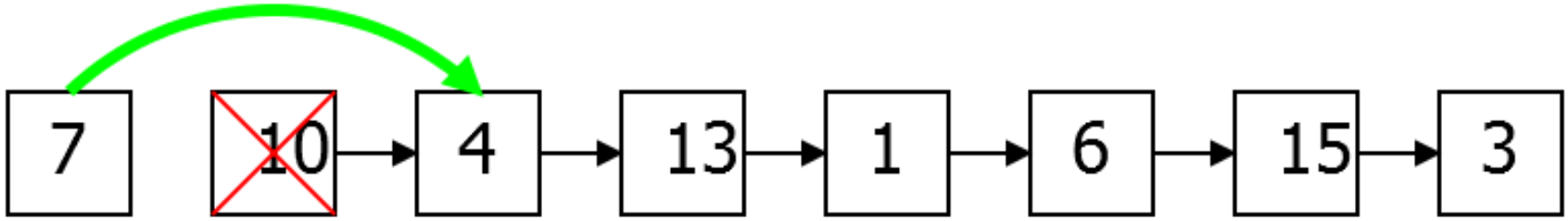
Linked List

- `RangeSearch`: $O(n)$ ×
- `NearestNeighbors`: $O(n)$ ×
- `Insert`: $O(1)$ ✓



Linked List

- `RangeSearch`: $O(n)$ ×
- `NearestNeighbors`: $O(n)$ ×
- `Insert`: $O(1)$ ✓
- `Delete`: $O(1)$ ✓



Need Something New

Problem

Previous data structures won't work. We need something new.