

Binary Search Trees: Splay Trees: Introduction

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Data Structures Fundamentals
Algorithms and Data Structures

Learning Objectives

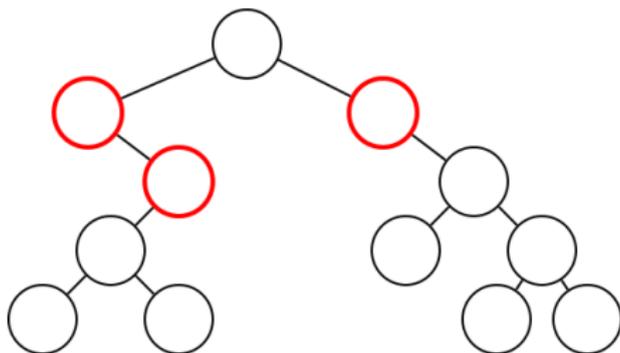
- Understand the motivation behind a splay tree.
- Implement the splay operation.

Non Uniform Inputs

- Search for random elements $O(\log(n))$
best possible.

Non Uniform Inputs

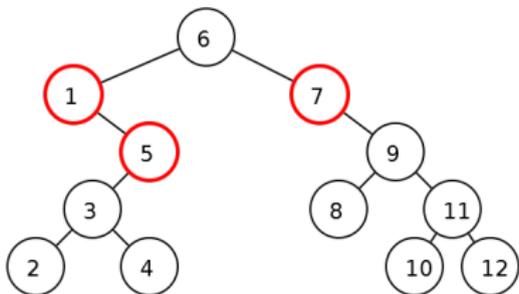
- Search for random elements $O(\log(n))$ best possible.
- If some items more frequent than others, can do better putting frequent queries near root.



Comparison

Trees.

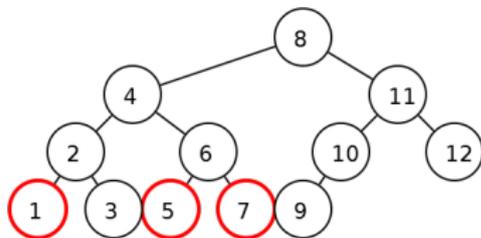
Unbalanced



Total

0

Balanced



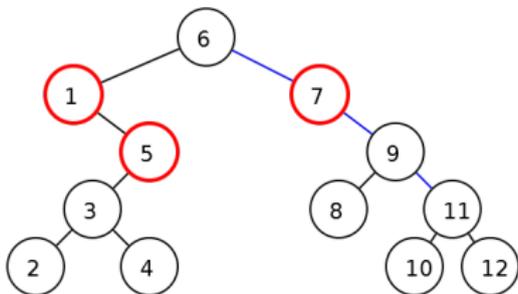
Total

0

Comparison

Find 11

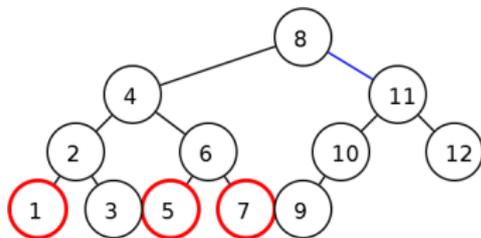
Unbalanced



Total

4

Balanced



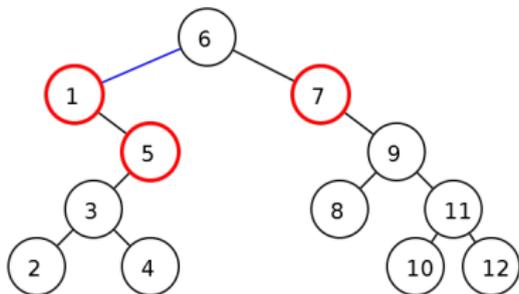
Total

2

Comparison

Find 1

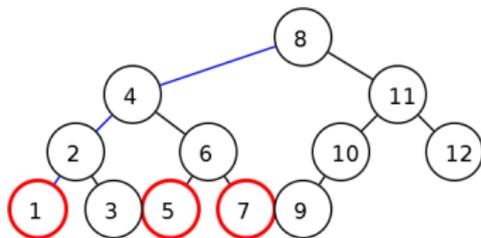
Unbalanced



Total

6

Balanced



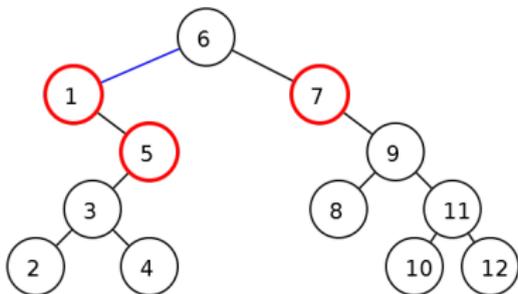
Total

6

Comparison

Find 1

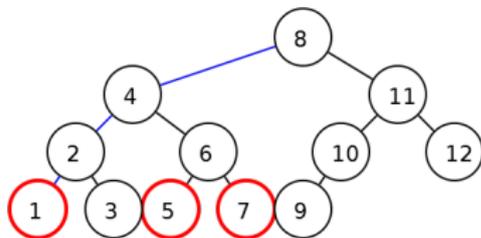
Unbalanced



Total

8

Balanced



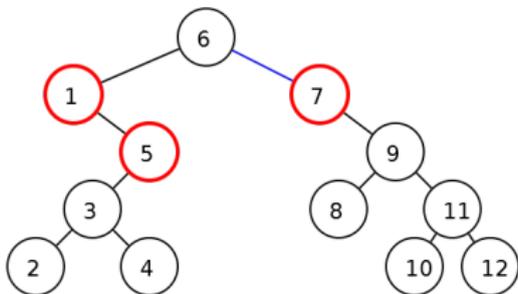
Total

10

Comparison

Find 7

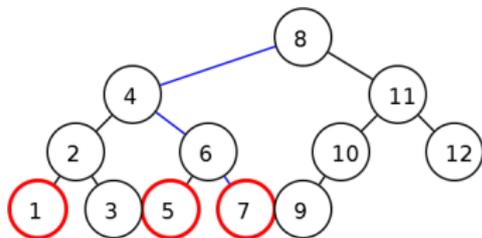
Unbalanced



Total

10

Balanced



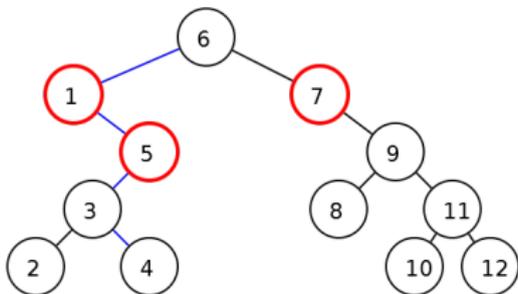
Total

14

Comparison

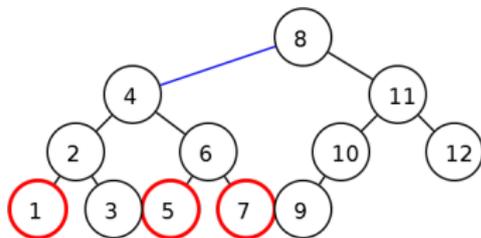
Find 4

Unbalanced



Total
15

Balanced

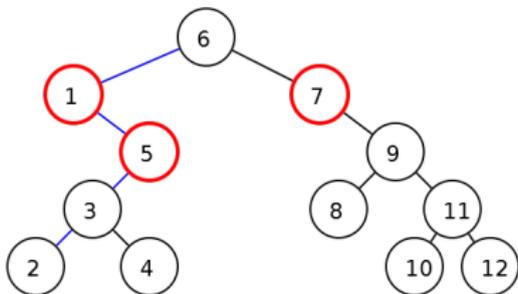


Total
16

Comparison

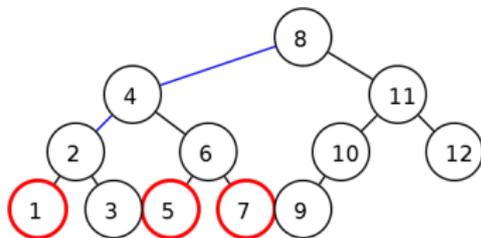
Find 2

Unbalanced



Total
20

Balanced

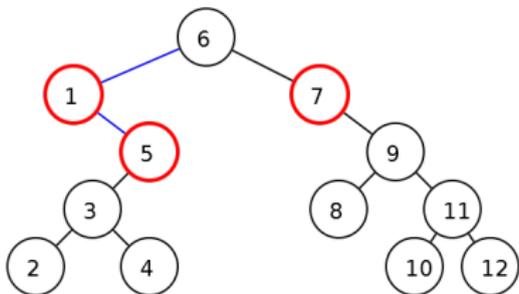


Total
19

Comparison

Find 5

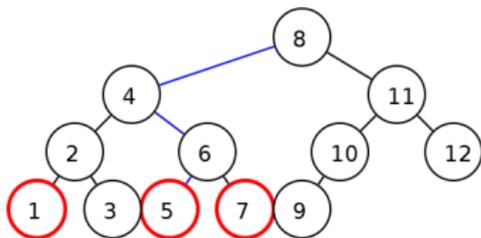
Unbalanced



Total

23

Balanced



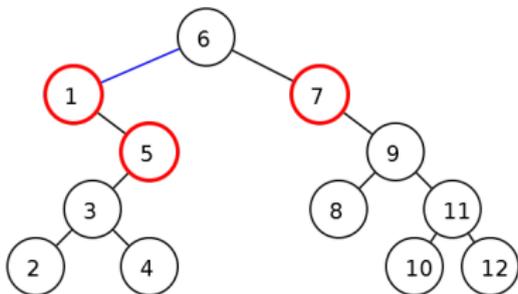
Total

23

Comparison

Find 1

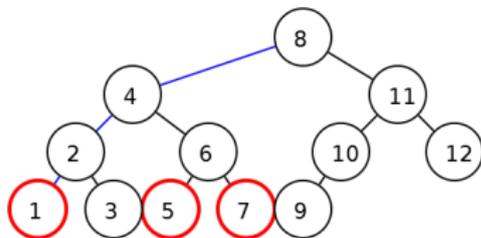
Unbalanced



Total

25

Balanced



Total

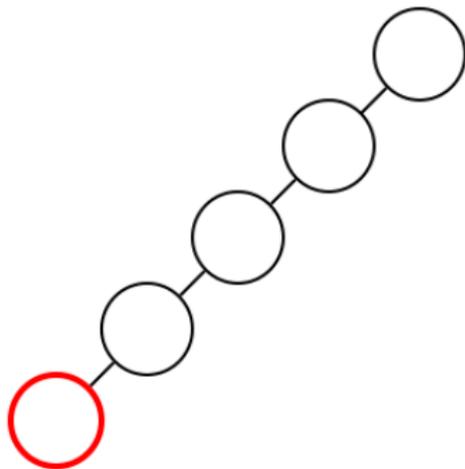
27

Idea

- Want common nodes near root.
- Don't know which those nodes will be.
- Bring the queried node to the root.

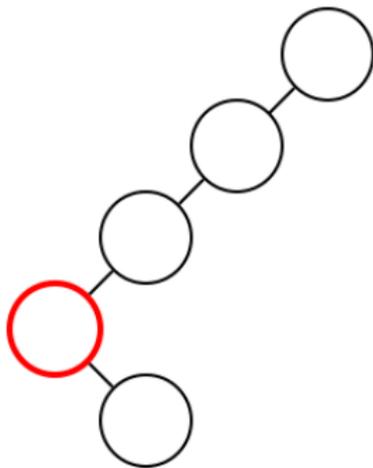
Simple Idea

Just rotate to top.



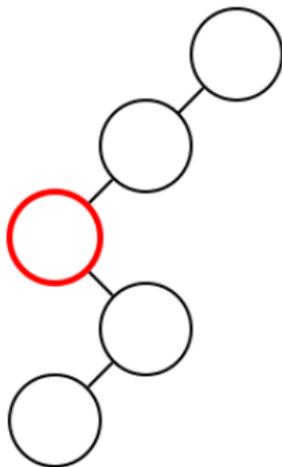
Simple Idea

Just rotate to top.



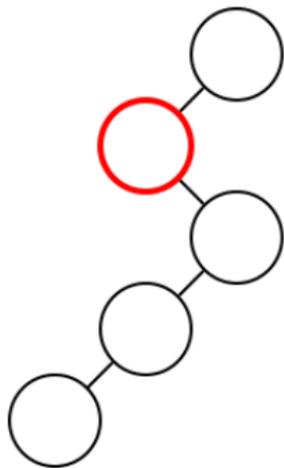
Simple Idea

Just rotate to top.



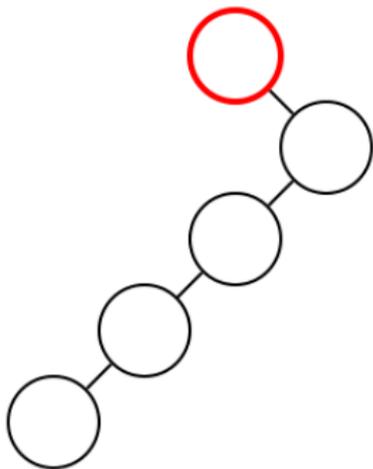
Simple Idea

Just rotate to top.



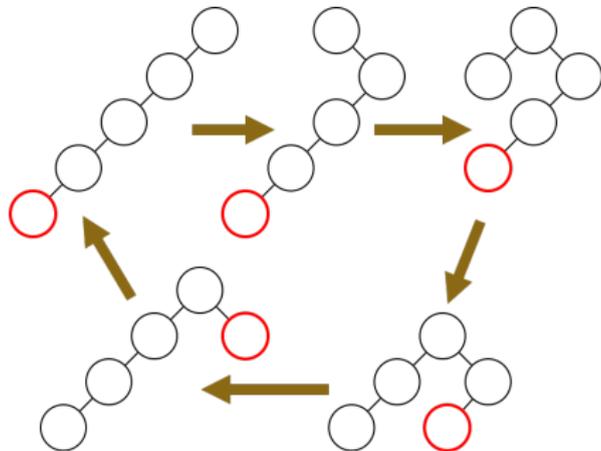
Simple Idea

Just rotate to top.



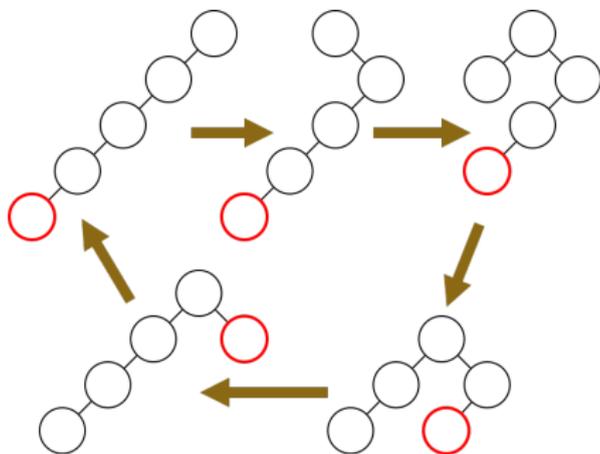
Loop

If you keep doing this you can get stuck in a loop.



Loop

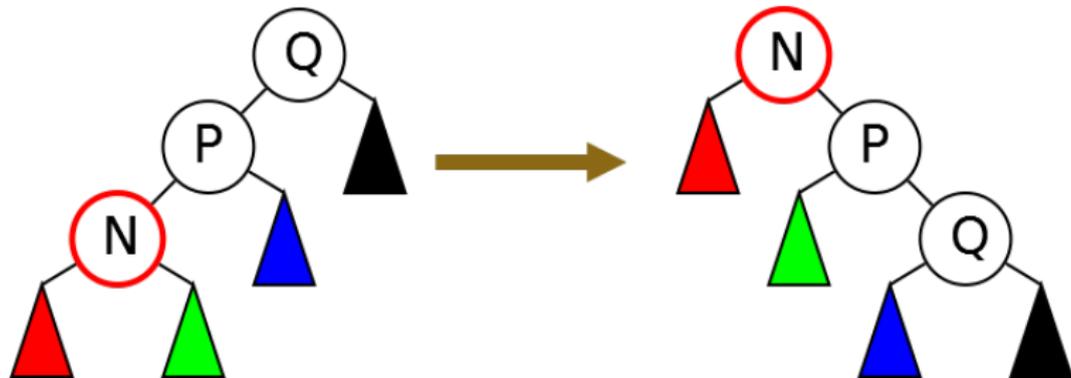
If you keep doing this you can get stuck in a loop.



$O(n^2)$ time for $O(n)$ operations. Need something better.

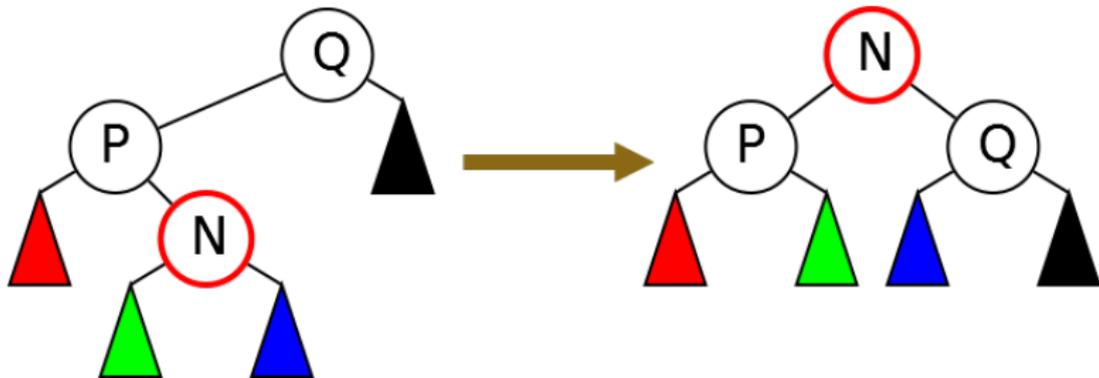
Modification

Zig-Zig



Modification

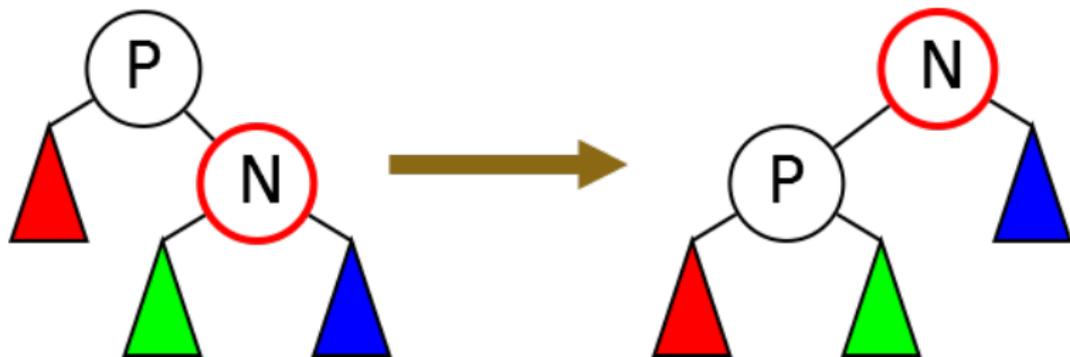
Zig-Zag



Modification

If just below root:

Zig



Splay

Splay(N)

Determine proper case

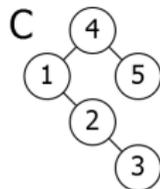
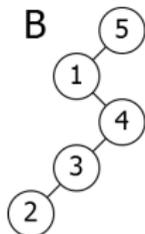
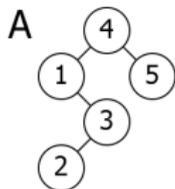
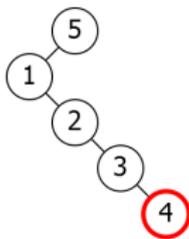
Apply Zig-Zig, Zig-Zag, or Zig as appropriate

if $N.Parent \neq \text{null}$:

 Splay(N)

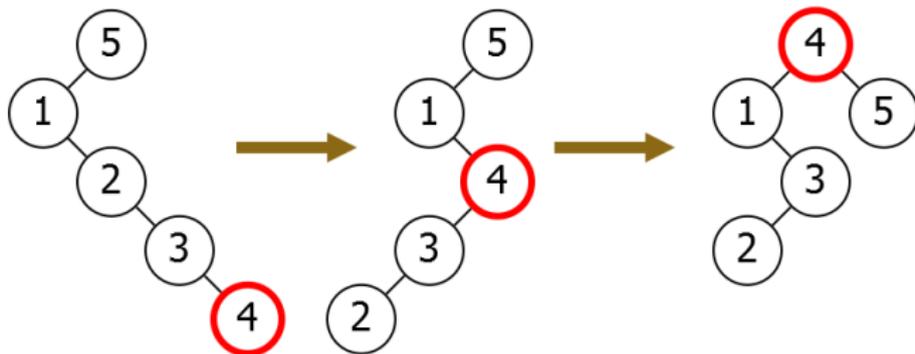
Problem

Which of the following is the result of playing the highlighted node?



Problem

Which of the following is the result of playing the highlighted node?



Next Time

How to use the splay operation to rebalance your tree.